Name: ______________________________ **Roll No.:** ____________________

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2017

Program:	**B. Tech. Chemical Engg. (spl. RP); CE-RP**	**Semester:**	**V**
Subject (Course):	**Numerical Methods in Chemical Engineering**	**Max. Marks:**	**100**
Course Code:	**MATH 311**	**Duration:**	**3 Hrs**
No. of page/s:	**0 + 3**		

In this **OPEN BOOK and NOTES EXAM**, you are allowed to have the textbook (S. K. Gupta, *Numerical Methods for Engineers*), ***all*** handouts provided, ***your own class-notes*** and your solutions to assignment problems

1. Show all ***intermediate steps*** of your answers (and not just the final answers) to earn marks
2. ***Please answer the questions in the sequence:*** **1, 2, 3.** You can do this by assigning, *a priori*, a few pages to each question, in the correct sequence. You may then answer the questions in whatever sequence you wish to, ***all parts in one place***
3. No student is allowed to leave the examination hall in the first hour of the exam

Section A: XXX

No questions here

Section B: ALL THREE QUESTIONS ARE COMPULSORY [Total 100 Marks]

Q.1 Consider the following set of coupled, non-linear ODE-BVPs with a single independent variable, x, and **two** dependent variables, $y(x)$ and $z(x)$, with $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$:

$$y'' + 3y = 3y^2 z$$

$$z'' + 4z = 2y + 6z^2$$

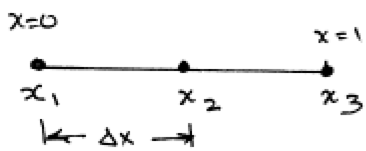
with the following BCs:

Continued…

$y'(0) = 5; \quad y(1) = 4$

$z(0) = 2; \quad z(1) = 6$

(i) Use $N = 2$ (i.e., $N + 1 = 3$) and the FD method with expressions correct to $O(\Delta x)^2$ to solve this problem (give the ***complete*** set of equations) (30)



(30 *Points*)

Q.2 Consider the explicit, third order Hermite ODE-IVP algorithm (given in Table 5.1):

$$y_{n+1} = -4y_n + 5y_{n-1} + h(4y'_n + 2y'_{n-1}) + O(h^3)$$

(a) Obtain the characteristic equation for this technique. (09)

(b) Obtain ***all*** the roots, μ_i, of this equation. (09)

(c) Identify which of these roots is the non-spurious root, μ_1. (09)

Hint: You may use the following expansions:

$(1 + x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \cdots$, and

$e^x = 1 + x \ ...$

(d) Now evaluate the numerical value of $|\mu_1|$ for $|h\lambda| = 0$ (08) (**35Points**)

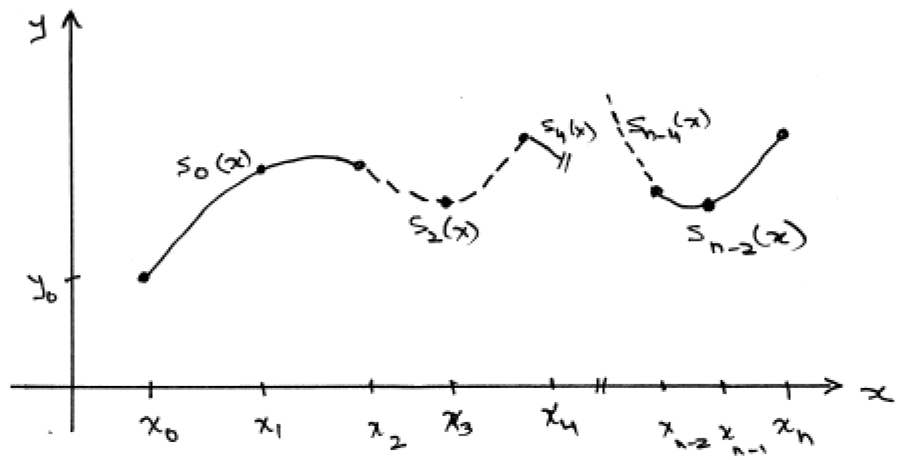
Q.3 Consider a ***NEW*** ***quadratic*** spline fit for a set of given points, $(x_0, y_0), (x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, with n being an ***even number***. You decide to fit ***each*** of the quadratic splines to three adjacent points as follows (see diagram below):

Continued…

$S_0(x)$: Points (x_0, y_0), (x_1, y_1) and (x_2, y_2),

$S_2(x)$: Points (x_2, y_2), (x_3, y_3) and (x_4, y_4), . . .

S_{n-2}(x): Points (x_{n-2}, y_{n-2}), (x_{n-1}, y_{n-1}) and (x_n, y_n).



Note again that each of the spines are quadratic (***not linear nor cubic***) and each passes through three points as discussed above (this is a stupid thing to do, but you still do it).

(a) Write down the ***most convenient*** equations for $S_i(x)$, i = 0, 2, 4, . . . , n - 4, n – 2, involving constants, $a_{i,j}$. (10)

(b) Write down ***all*** the equations for determining the constants, $a_{i,j}$. Since the algebra involved is quite a bit, do ***not*** solve these equations (except the ones which are extremely simple). (12)

(c) Comment on the continuity of the ***values*** of $S_i(x)$ and $S_{i+2}(x)$ at the end-points, i.e., at x = x_{i+2}, as well as the values of $S_i'(x)$ and $S_i''(x)$ at these points. (13) (**35 Points**)

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