UNIVERSITY OF PETROLEUM & ENERGY STUDIES
DEHRADUN
End Semester Examination – December, 2017

Program/course: M.TECH | **Semester – I**
Subject: Numerical Methods for Nuclear Engineering | **Max. Marks: 100**
Code: MATH-7004 | **Duration: 3 Hrs**
No. of page/s: 2

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt all questions from **Section C** (each carrying 20 marks).

Section A

1. Prove that $E \equiv e^{hD}$, where D is the differential operator, $E$is shift operator and h is the interval of differencing. [4]

2. Derive Newton-Raphson iteration formula for finding the positive square root of a real number a. [4]

3. Explain by graph how Fixed-point iteration formula $x_{n+1} = \varphi(x_n), n \geq 1$ converges for the condition $|\varphi'(x)| < 1$ for some $x \in (a,b)$; where (a,b) is the interval bracketing the root of $f(x) = 0$. [4]

4. Consider $\frac{dy}{dx} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ with $y(0) = 1$. Find the approximate value of $y(1)$ using Taylor's series method, given $h = 1$. [4]

5. If the fourth order divided difference of $f(x) = kx^4 + 5x^3 + 3x + 2, k \in R$, at the points $0.1, 0.2, 0.3, 0.4, 0.5$ is 5, then find the value of k. [4]

Section B

All questions are compulsory.

6. The fourth order Runge−¿Kutta method

$$u_{j+1} = u_j + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

is used to solve the initial value problem:

$$\frac{du}{dt} = u, u(0) = \alpha.$$

If $u(1) = 1$ is obtained by taking the step size $h = 1$,then find the value of α . [10]

7. Solve $u_t = 5u_{xx}$ with $u(0,t)=0; u(5,t)=60$ and $u(x,0)=\begin{cases} 20x \; for \; 0<x\le 3 \\ 60 \;\; for \; 3<x\le 5 \end{cases}$; for five time steps taking $h=1$ by using Bender-Schmidt method. [10]

8. Let α be a root of $f(x)=0$ which is equivalent to $x=\varphi(x)$. Let I be the interval containing α and $|\varphi'(x)|<1$ for all x in I. Using this scheme, what is the approximate value of α correct to two decimal places if $f(x)=\cos x - 3x + 1$ and $I=(0,1)$. [10]

9. Consider a function $\varphi(x)=(x)$, where (x) is the integer closest to real number x(if there are two such numbers, select the greater one). Apply numerical integration to evaluate $\int_1^3 \varphi(x)\,dx$, by dividing the interval $(1,3)$ into 10 equal sub- intervals. [10]

OR

Solve $u_t = u_{xx}$ with $u(x,0)=0; u(0,t)=0$ and $u(1,t)=1$. Compute u for $t=1/8$ in two time steps, using Crank-Nicholson's method. [10]

Section C

All questions are compulsory.

10. Let $f(x)$ be a differentiable function such that $\frac{d^3 f}{dx^3}=1$ for all $x\in[0,3]$. If $p(x)$ is the quadratic polynomial which interpolates $f(x)$ at $x=0, x=2$ and $x=3$, then find the value of $f(1)-p(1)$. [20]

OR

a. Given the differential equation: $(y^2+1)\,dy = x^2\,dx$ with the initial condition $y=0$ when $x=0$. Use Picard's method to obtain y for $x=0.25$ correct to three decimal places. [10]

b. Consider the initial value problem:

$$\frac{dy}{dx}=f(x,y),\; y(x_0)=y_0.$$

The aim is to compute the value of $y_1 = y(x_1)$, where $x_1 = x_0 + h\,(h>0)$. At $x=x_1$,if the value of y_1 is obtained by the straight line passing through (x_0, y_0) and having slope equal to the slope of the curve $y(x)$ at $x=x_0$, then identify the method and calculate $y(0.1)$ if $f(x,y)=\frac{y-x}{y+x}$ and $x_0=0, y_0=1, h=0.02$. [10]

11. The matrix $A=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ can be decomposed uniquely into the product $A=LU$, where

$$L=\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U=\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{23} \end{bmatrix}.$$

Let $x \in R^3 \wedge b = [1,1,1]^t$.

Find the solution of the system $Ax = b$ where $x = [x_1, x_2, x_3]^t$ [20]

OR

Solve the system of equations

$$x + y + z = 1, x + 2y + 3z = 1, x + 2y + 3z = 3$$

by Choleski decomposition method. [20]