Name: **Enrolment No:**	UPES

End Semester Examination, December 2017

Course: MATH 7001- Applied Mathematics in Petroleum Engineering -I

Programme: M. Tech. (Petroleum Engineering)
Semester: I (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

Section A
(Attempt all questions)

1.	If $y = x^3 + x^2 - 2x + 1$, calculate values of y for $x = 0,1,2,3,4,5$ and form the difference table.	**[4]**	**CO1**
2.	Evaluate $\int_0^4 e^x dx$, by Simpson's 1/3 rule, using data $e = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$.	**[4]**	**CO2**
3.	Find the positive root of the equation $x - \cos x = 0$ using bisection method.	**[4]**	**CO3**
4.	Estimate the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ using the Gerschgorin bounds.	**[4]**	**CO4**
5.	Use Picard method to solve the equation $y' = x - y$ subject to the condition $y = 1$ when $x = 0$.	**[4]**	**CO5**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	The following table gives the population of a town Kosikalan during the last six censuses. Estimate the population in 1913 by Newton's forward difference formula Year: 1911 1921 1931 1941 1951 1961 Population: 12 15 20 27 39 52 (in thousands)	**[8]**	**CO1**
7.	Find the root of the equation $x^6 - x^4 - x^3 - 1 = 0$ which lies in the interval (1.4, 1.5) correct to four decimal places using Regula- Falsi method.	**[8]**	**CO3**

8.	Solve the equation $\frac{dy}{dx}=x+y$ with initial condition $y(0)=1$ by Runge – Kutta method, from $x=0$ to $x=0.1$ with $h=0.1$.	[8]	CO5
9.	Solve $u_{xx}+u_{yy}=0$ in $0\le x\le 4, 0\le y\le 4$, given that $u(0,y)=0, u(4,y)=8+2y$, $u(x,0)=\frac{x^2}{2}$, and $u(x,4)=x^2$. Take $h=k=1$ and obtain the result correct to one decimal place.	[8]	CO6
10.	Find the solution of $\frac{dy}{dx}=\frac{y-x}{y+x}, y(0)=1$. Find y approximately for $x=0.6$ by Euler's method. Take $h=0.1$. **OR** Use Taylor's series method to solve $\frac{dy}{dx}=x+y$; $y(1)=0$ numerically upto $x=1.2$ with $h=0.1$. Compare the final result with the value of explicit solution.	[8]	CO5
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Using Jacobi method find all the eigenvalues and the corresponding eigenvectors of the matrix $$A=\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}.$$	[10]	CO4
11.B	The following are the measurements t made on a curve recorded by the oscillograph representing a change of current i due to a change in the conditions of an electric current. t : 1.2, 2.0, 2.5, 3.0 i : 1.36, 0.58, 0.34, 0.20 Using Lagrange's formula, find i at $t=1.6$.	[10]	CO1
12.A	A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below:	[10]	CO2

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<tr><td>h (in meters):</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr>
<tr><td>A (in sq. meters):</td><td>950</td><td>1070</td><td>1200</td><td>1350</td><td>1530</td></tr>
</table>
If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dh}{dt} = -\frac{48}{A}\sqrt{h}$.
Estimate the time taken for the water level to fall from 14 to 10 m above the sluices.
OR
The table below gives the results of an observation; θ is the observed temperature in degrees centigrade of a vessel of cooling water; t is the in minutes from the beginning of observation.
<table>
<tr><td>t :</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr>
<tr><td>θ :</td><td>85.3</td><td>74.5</td><td>67.0</td><td>60.5</td><td>54.3</td></tr>
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Find the approximate rate of cooling at $t = 3$ and 3.5.
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<td>12.B</td>
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Solve the heat conduction problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to conditions $u(x,0) = \sin \pi x,$ $0 \le x \le 1$, and $u(0,t) = u(1,t) = 0$, using Schmidt method and Crank – Nicolson method, taking $h = 1/3,\ k = 1/36$.
OR
Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0$, $y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length = 1.
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<td>[10]</td>
<td>CO6</td>
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