Name:

Enrolment No:

UPES

End Semester Examination, Dec 2017

Course: MATH 7002 – Advanced Mathematics

Programme: M. Tech Rotating Equipment **Semester: I (ODD-2017-18)**

Time: 03 hrs. **Max. Marks:100**

Instructions:

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Find a positive root of $cosx + sinx - 1 = 0$ correct to 3 decimal places using bisection method.	**[4]**	**CO2**
2.	Solve $x^2\, e^{\frac{-x}{2}} = 1$ using regula-falsi method.	**[4]**	**CO2**
3.	Evaluate $\Delta^2 \cos 2x$, where Δ is called forward difference operator.	**[4]**	**CO1**
4.	Discretize the differential equation $y'' + \frac{4x}{1+x^2}y' + \frac{2}{1+x^2}y = 0,\ y(0) = 1,\ y(2) = 0.2$ using forward, backward and central difference approximations.	**[4]**	**CO3**
5.	Derive the least square equations for fitting a curve of the type $Y = aX + \frac{b}{X}$ to a set of n points $(x_i, y_i); i = 1, 2, 3, \ldots, n$.	**[4]**	**CO5**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned from various values of time t (in seconds) t: 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2 θ: 0, 0.12, 0.49, 1.12, 2.02, 3.20, 4.67 Calculate the angular velocity and angular acceleration of the rod at $t = 0.1$ sec.	**[8]**	**CO1**
7.	Consider the data function $f(x) = \frac{e^x\, sinx}{1+x^2}$ and answer the following: (a). prepare $[x_i,\ f(x_i)]$ table by taking $h = \frac{1}{7}$ in $[0,\ 1]$. (b). Evaluate $\int_0^1 f(x)dx$ using Simpson's (1/3) rule from $x = 0$ to $x = \frac{6}{7}$ and then use Trapezoidal rule from $x = \frac{6}{7}$ to $x = 1$ with the help of tabular values obtained above.	**[8]**	**CO1**

<table>
<tr><td>8.</td><td>Find the solution of the system of equations $45x_1 + 2x_2 + 3x_3 = 58; 5x_1 + x_2 + 20x_3 = 67$
$-3x_1 + 22x_2 + 2x_3 = 47$, correct to two decimal places, using Gauss-Seidel iteration method.</td><td>[8]</td><td>CO2</td></tr>
<tr><td>9.</td><td>The ranks of same 16 students in Mathematics and Physics are as follows. Two numbers within brackets denote the ranks of the students in Mathematics and Physics.
$(1,1),(2,10),(3,3),(4,4),(5,5),(6,7),(7,2),(8,6),$
$(9,8),(10,11),(11,15),(12,9),(13,14),(14,12),(15,16),(16,13)$
Calculate the rank correlation coefficient for proficiencies of this group in Mathematics and Physics.</td><td>[8]</td><td>CO5</td></tr>
<tr><td>10.</td><td>By the method of least squares ,find the curve $y = ax + bx^2$ that best fits the following data:
<table>
<tr><td>x</td><td>1</td><td>2.5</td><td>5</td><td>9</td><td>10</td></tr>
<tr><td>y</td><td>18</td><td>7.1</td><td>8.9</td><td>11</td><td>13</td></tr>
</table>
OR
The mean yield for one-acre plot is 662 kilos with a s.d. 32 kilos. Assuming normal distribution, how many, one-acre plots in a batch of 1000 plots would you expect to have yield (i) over 700 kilos, (ii) below 650 kilos, and (iii) what is the lowest yield of the best 100 plots?
Given that $P(0 \le z \le 1.19) = 0.3830; P(0 \le z \le 0.38) = 0.1480; P(0 \le z \le 1.28) = 0.4$</td><td>[8]</td><td>CO5</td></tr>
<tr><td colspan="4">SECTION C
(Q11 is compulsory and Q12 has internal choice)</td></tr>
<tr><td>11.A</td><td>Use the Runge-Kutta 4th order method with step length $(h) = 0.5$ to obtain an approximation to $y(1)$ for the solution of $\frac{dy}{dx} = \frac{x^2}{1+y^2}$; $y(0) = 0$</td><td>[10]</td><td>CO3</td></tr>
<tr><td>11.B</td><td>In the following table are recorded data showing the test scores by salesmen on an intelligence test and their weekly sales:
<table>
<tr><td>Salesmen</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Test scores</td><td>64</td><td>71</td><td>53</td><td>67</td><td>55</td><td>58</td><td>77</td><td>57</td><td>56</td><td>51</td></tr>
<tr><td>Sales (in 1000s)</td><td>8.1</td><td>6.0</td><td>5.5</td><td>5.1</td><td>3.5</td><td>2.8</td><td>5.5</td><td>3.7</td><td>7.0</td><td>4.3</td></tr>
</table>
Calculate the regression line of sales on test scores and estimate the most probable weekly sales volume if a salesman makes a score of 70.</td><td>[10]</td><td>CO5</td></tr>
</table>

12.	Solve the boundary value problem $u_{xx} + u_{yy} = x + y + 1, 0 \le x \le 1, 0 \le y \le 1, u = 0$ on the boundary numerically using five point formula and Liebmann iteration, with mesh length $h = \frac{1}{3}$. Obtain the results correct to three decimal places. **OR** Solve the equation $u_{xx} = u_t$ subjected to $u(0,\ t) = 0,\ u(1,\ t) = 0,\ t > 0$ and $u(x,\ 0) = sin(\pi x),\ 0 \le x \le 1$ (a) using Crank-Nicolson method with $h = \frac{1}{3},\ k = \frac{1}{36}$ for one time step. (b) using Bender-Schmidt method with $h = \frac{1}{3},\ \lambda = \frac{1}{2}$ for two time steps.	**[20]** **[10]** **[10]**	**CO4**