Name: **Enrolment No:**	UPES

End Semester Examination, Dec 2017
Course: MATH 222-Mathematics-III
Programme: B. Tech (ADE, APE Gas, CERP, ME MD, ME MSNT, ME PE, ME TE, Mechanical, Mechatronics)
Semester: III (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Solve the first order nonlinear equation $p = \cot(px - y)$ where $p = \frac{dy}{dx}$.	**[4]**	**CO1**
2.	If functions $f(z)$ and $g(z)$ are analytic at z_0 and $f(z_0) = 0 = g(z_0)$ but $g'(z_0) \neq 0$, then prove that $lim_{z \to z_0} \frac{f(z)}{g(z)} = \frac{f'(z_0)}{g'(z_0)}$.	**[4]**	**CO2**
3.	Evaluate $\oint_{\lvert z \rvert =1} \frac{z^2-9}{coshz}\, dz$.	**[4]**	**CO3**
4.	Show that $z = \infty$ (point at infinity) is an isolated essential singularity of the function $f(z) \equiv sinz - cosz$.	**[4]**	**CO3**
5.	Form a partial differential equation by eliminating the arbitrary constants a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	**[4]**	**CO4**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Show that the equation $(3y + 5xy^3)dx + (3x + 4x^2y^2)dy = 0$ is not exact. Find an integrating factor $\mu(x, y)$ to the given equation and hence solve the equation.	**[8]**	**CO1**
7.	Find the real general solution of the non-homogeneous liner system $x' = Ax + f(\theta)$ for the given matrix $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$ and $f(\theta) = \begin{bmatrix} tan\theta \\ 1 \end{bmatrix}$ where $x' = \frac{dx}{d\theta}$.	**[8]**	**CO1**

8.	Find the integral of the function $f(z) = (z^2 - z + 2)$, between two points $z = 0$ and $z = 1 + i$ along two different paths and hence compare the results with direct integration.	**[8]**	**CO2**
9.	Evaluate $\int_{\lvert z\rvert=1} \frac{ze^z}{(4z+\pi i)^2}\, dz$ using Cauchy's integral formula.	**[8]**	**CO2**
10.	Find all the singularities of $g(z) = \frac{\sin(z^2)}{z^5 \sin(z)}$ in $\mathbb{C}$ and hence classify their nature. Also, find the residue at each isolated singularity. **OR** Find all the singularities of $g(z) = \frac{z-1}{\exp\left(\frac{2\pi i}{z}\right) - 1}$ in $\mathbb{C}$ where $\exp\left(\frac{2\pi i}{z}\right) = e^{\frac{2\pi i}{z}}$.	**[8]**	**CO3**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Apply calculus of residues to prove that $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \frac{\pi}{2}$.	**[10]**	**CO3**
11.B	A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x,t)$.	**[10]**	**CO4**
12.A	Expand $h(z) = \frac{7z-3}{z(z-1)}$ in a Laurent series valid for the annular domains (i) $0 < \lvert z\rvert < 1$ and (ii) $0 < \lvert z-1\rvert < 1$. Find the residue of $h(z)$ at $z = 0$ and $z = 1$ from the obtained Laurent series. **OR** Apply calculus of residues to prove that Cauchy principal value of $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9}\, dx = \frac{5\pi}{12}$.	**[10]**	**CO3**
12.B	Solve $(D_x^2 - 2D_xD_y - 3D_y^2)z = e^{y-x}\sin(y-2x)$ where $D_x \equiv \frac{\partial}{\partial x}$ and $D_y \equiv \frac{\partial}{\partial y}$. **OR** Form a partial differential equation by eliminating the arbitrary functions f, g and ϕ from $z = f(x-at) + x\,g(x-at) + x^2\,\phi(x-at)$.	**[10]**	**CO4**

Name: **Enrolment No:**	UPES

End Semester Examination, Dec 2017
Course: MATH 222 – Mathematics III
Programme: B.Tech. (ADE, APE Gas, CERP, ME MD, ME MSNT, ME PE, ME TE, Mechanical, Mechatronics)
Semester: III (ODD-2017-18)
Time: 03 hrs. **Max. Marks:100**

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Solve the system of differential equation $$\frac{dx}{dt} = 2x + 3y$$ $$\frac{dy}{dt} = 2x + y$$	**[4]**	**CO1**
2.	Evaluate $\oint_C \frac{e^{3iz}}{(z+\pi i)^3} dz$, where C is the circle $\lvert z - \pi \rvert = 3.2$.	**[4]**	**CO2**
3.	Find the radius of convergence of the power series: $$\sum_{n=0}^{\infty} a_n z^n, \text{ where } a_n = \begin{cases} 2^n, & n \text{ is even} \\ 2^{-n}, & n \text{ is odd} \end{cases}$$	**[4]**	**CO3**
4.	Discuss the nature of all the singularities of the function $f(z) = \frac{z^2-\pi^2}{\sin z}$.	**[4]**	**CO3**
5.	Solve the partial differential equation: $$\frac{\partial^3 z}{\partial x^3} - 5\frac{\partial^3 z}{\partial x^2 \partial y} + 5\frac{\partial^3 z}{\partial x \partial y^2} + 3\frac{\partial^3 z}{\partial y^3} = 0$$	**[4]**	**CO4**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6.	Solve the equation: $$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right)dx + \frac{1}{4}(1 + y^2)xdy = 0$$	**[8]**	**CO1**
7.	Let f denote an arbitrary function and $p = \frac{dy}{dx}$ then find the non-singular solution of non-linear differential equation $y = 2px + f(xp^2)$.	**[8]**	**CO1**
8.	Using Cauchy's integral formula to evaluate $\oint_C \frac{\sinh(z^{2017})}{z^3} dz$, where C is the circle $z = e^{i\theta}, 0 \le \theta \le 4\pi$ oriented counterclockwise.	**[8]**	**CO2**

9.	Let $f(z) = u + iv$ and $g(z) = v + iu$ be analytic functions for all z. Let $f(0) = 1$ and $g(0) = i$. Obtain the value of $h(z)$ at $z = 1 + i$ where $h(z) = f'(z) + g'(z) + 2f(z)g(z)$.	**[8]**	**CO2**
10.	Evaluate the integral $\oint_C \frac{z^{99}}{z^{100}-1} dz$, where C is the circle $\|z\| = 100$. **OR** Using the calculus of residues, evaluate the following integral: $$\int_{-\infty}^{\infty} \frac{1}{x^6 + 64} dx$$	**[8]**	**CO3**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice)**			
11.A	Determine the poles of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ and the residue at each pole. Hence evaluate $\oint_C f(z)dz$ where C is the circle $\|z\| = 10$.	**[10]**	**CO3**
11.B	Solve the partial differential equation $\frac{\partial^2 z}{\partial x \partial y} = xy$ subject to the conditions $z(x, 0) = e^x + 1$ and $z(0, y) = e^y + 1$.	**[10]**	**CO4**
12.A	Apply Calculus of residues to prove that $$\int_0^{2\pi} \frac{d\emptyset}{(a+b\cos\emptyset)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}} \text{ where } a > 0, b > 0, a > b$$ **OR** Obtain the Taylor's series expansion of the function $f(z) = \frac{1}{z^2+4}$ about the point $z = -i$. Also, find the region of convergence.	**[10]**	**CO3**
12.B	Let f be a differentiable function of two variables x and y. Verify that the function $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ is a solution of the partial differential equation $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$. **OR** A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.	**[10]**	**CO4**