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# UNIVERSITY OF PETROLEUM & ENERGY STUDIES

**DEHRADUN**

**End Semester Examination - December, 2017**

**Program/Course:** B. Tech. Chemical Engineering (RP) **Semester–VII**
**Subject:** Process Optimization **Maximum Marks : 100**
**Code:** CHEG322 **Durations : 3 Hrs.**
**No. of Pages:** 2(Two)

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**Section-A (4x15 = 60 marks)**

Answer all **Four** Questions

1. a) Given $\ln n! = n \ln n - n$ for $n \gg 1$, show that $n! = \left(\frac{n}{e}\right)^n$ and $\frac{d \ln n!}{dn} = \ln n$ (5)

   b) If $a_j > 0$ for $j = 1, 2, 3 \cdots n$, then prove that (10)

$$\sum_{j=1}^{n} a_j \sum_{j=1}^{n} \frac{1}{a_j} \geq n^2$$

2. a) If $\boldsymbol{a_1, a_2, a_3}$ & $\boldsymbol{b_1, b_2, b_3} > 0$ and $\boldsymbol{(a_1 + a_2 + a_3)^2 = b_1 + b_2 + b_3}$, then prove that (10)

$$\frac{a_1^2}{b_1} + \frac{a_2^2}{b_2} + \frac{a_3^2}{b_3} \geq 1$$

   **Hint:** use Cauchy Inequality for the number pair $\left(\sqrt{\frac{a_i^2}{b_i}}, \sqrt{b_i}\right)$.

   b) For real $x$, prove that $\boldsymbol{\cosh(x) \geq 2}$ (5)

3. a) Show that (10)

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx = 1$$

   **Hint:** use the relation: $\Gamma(\frac{1}{2}) = \sqrt{\pi} = 2\int_{0}^{\infty} \exp(-x^2) dx$

   b) For $x > 0$, prove that $\boldsymbol{\ln x \leq x - 1}$ (5)

4. A triangle **ABC** is inscribed in a circle with its base **AB** as the diameter, with length **2** (arbitrary unit), of a circle. The points **A** and **B** are the two vertices of the triangle **ABC**. The other vertex **C** slides along the perimeter of the circle so that the area of the triangle **ABC** changes. Find the length of the sides **AC** and **BC** when the area of the triangle **ABC** is maximum. What is the ratio between the area of the semi-circle and that of the triangle. (15)

   **Hint:** Take the two end points of the base (as $x$-axis) of the triangle as $\boldsymbol{(-1, 0)}$ & $\boldsymbol{(1, 0)}$ and the equation of the circle as $\boldsymbol{x^2 + y^2 = 1}$.

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**Section-B (2x20 = 40 marks)**

Answer any **Two** Questions

**5.** **a)** Maximize the Objective Function ($\boldsymbol{P}$) given by (15)

$$\boldsymbol{P = 3x + 4y}$$

subject to

$$\boldsymbol{2.5x + y \leq 20}$$
$$\boldsymbol{3x + 3y \leq 30}$$
$$\boldsymbol{x + 2y \leq 16}$$
$$\boldsymbol{x \geq 0}$$
$$\boldsymbol{y \geq 0}$$

using Linear Programming Technique.

**Hint**: Enumeration Method can be used to avoid solving by graphical method.

**b)** Write the expression for the Taylor Series expansion of "cosh $\boldsymbol{x}$" up to the fourth term. (5)

**Hint**: $\cosh \boldsymbol{x} = \frac{1}{2}(e^x + e^{-x})$

**6.** **a)** A Restaurant in Panditwari, Dehradun, has been doing business since last five (5) months, and has recorded monthly profit (in thousands of Rupee) as follows: (10)

| **Month** ($t$) | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **Profit**($y = f(t)$) | **24** | **28** | **31** | **36** | **39** |

Applying the Linear Regression (LR) Analysis, predict the profit at the end of the sixth (6th) month. What are the units of the two parameters that are to be determined from the LR?

**b)** For a triangle ABC, prove that

$$SinA \cdot SinB \cdot SinC \leq \frac{3\sqrt{3}}{8}$$

hint: $\boldsymbol{f(x) = \ln Sinx}$ (10)

**7.** A consecutive **first** order reaction is occurring in a Continuous Stirred Tank Reactor (CSTR) of volume $\boldsymbol{V}$ at **steady state** isothermally, with the following scheme

$$\mathbf{A} \xrightarrow{k_1} \mathbf{B} \xrightarrow{k_2} \mathbf{C}$$

where $k_1$ and $k_2$ are the respective rate constants. The volumetric flow rate and the inlet concentration of the reactant **A**, are given by $Q$ and $C_{A_{in}}$ respectively, with $\boldsymbol{\tau}$ ($= \frac{\boldsymbol{V}}{\boldsymbol{Q}}$) as the residence time.

- Formulate and solve the governing equations to determine the outlet concentrations, $\boldsymbol{C}_{\mathrm{A_o}}$, $\boldsymbol{C}_{\mathrm{B_o}}$ and the $\boldsymbol{C}_{\mathrm{C_o}}$ for the A, B and C respectively, as function of residence time. (10)
- Show that the optimum residence time $\boldsymbol{\tau_{opt}}$, for obtaining the maximum concentration of **B**, $\boldsymbol{C}_{\mathrm{B_o}}^{\mathbf{max}}$, is given by (5)

$$\boldsymbol{\tau_{opt} = \frac{1}{\sqrt{k_1 k_2}}}$$

- Write the expression for the maximum concentration of **B**, $\boldsymbol{C}_{\mathrm{B_o}}^{\mathbf{max}}$, in terms of the optimum residence time, $\boldsymbol{\tau_{opt}}$. (5)