Roll No: ______________________________

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2019

Programme: BSc (Hons) Mathematics	**Semester – II**
Course Name: Differential Equations	**Max. Marks : 100**
Course Code: MATH 1031	**Duration : 3 Hrs**
No. of page/s:2	

Section A

(Attempt all questions)

		MARKS	
1.	Investigate the behavior of the solution of differential equation $\frac{dx}{dt}=\frac{-1}{10}\left(x^2-10x+9\right); x(0)=x_0$.	[4]	CO4
2.	Solve $\frac{dy}{dx}+secx\, y=tanx$	[4]	CO2
3.	Find an integrating factor of the differential equation $\left(x y^2-e^{\frac{1}{x^3}}\right)dx-x^2 ydy=0$	[4]	CO2
4.	Find the nature of solution of the differential equation $\frac{dy}{dx}=\frac{x^2}{1+y^2}$	[4]	CO1
5.	Let $f(D)y=e^{ax}$ be a linear n^{th}order differential equation then show that the particular integral $\frac{1}{f(D)}e^{ax}=\frac{1}{f(a)}e^{ax}$ provided $f(a)\neq 0$.	[4]	CO3

SECTION B

(All questions are compulsory, Q10 has internal choice)

6.	Find the equilibrium solutions of the autonomous equation $y'=y^2(1-y^2)$ and hence determine their stability.	[08]	CO5
7.	A lake of constant volume Vcontains at time $t$an amount $M(t)$of pollutant evenly distributed throughout the lake. Suppose water-containing concentration $c(t)$of pollutant enters the lake at a rate $F$and water leaves the lake at the same rate. Find a differential equation that models this process and determine the concentration of pollutant with $c(0)=c_0$.	[08]	CO4
8.	Solve $x\left(x^2+1\right)\frac{dy}{dx}=y\left(1-x^2\right)+x^3\log_e x$	[08]	CO2

9.	Explain some characteristics of mathematical models.	**[08]**	**CO4**
10.	Solve $\frac{d^2 y}{d x^2}+a^2 y=x\cos(ax)$ **OR** Solve $(D^4+2D^2+1)y=x^2 cosx$	**[08]**	**CO3**
	SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice**		
11. A	Consider the following system: $x'=-2x-y-2z;\ y'=-4x-5y+2z;\ z'=-5x-y+z$ Determine the stability of the equilibrium point the origin.	**[10]**	**CO5**
11.B	Solve the Cauchy-Euler equation $x^3\frac{d^3 y}{d x^3}-x^2\frac{d^2 y}{d x^2}+2x\frac{dy}{dx}-2y=x^3+3x$	**[10]**	**CO3**
12. A	Consider the differential equation $y=2px-p^2$ where $p=\frac{dy}{dx}$ (i) Find a one-parameter family of solutions (ii) Find an extra solution (if exists) that is not a member of the one-parameter family found in part (i). **OR** Find the general solution and singular solution (if exists) of the differential equation $p^3-4xyp+8y^2=0$ where $p=\frac{dy}{dx}$.	**[10]**	**CO2**
12.B	Solve $y''+y=4x+10\sin(x)$ using method of undetermined coefficients. **OR** Solve $y''-4y'+4y=(x+1)e^{2x}$ using variation of parameters method.	**[10]**	**CO3**

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Course Name: Differential Equations **Max. Marks : 100**
Course Code: MATH 1031 **Duration : 3 Hrs**
No. of page/s:2

Section A
(Attempt all questions)

		MARKS	
1.	Investigate the behavior of the solution of differential equation $\frac{dy}{dx}=ry$.	[4]	CO4
2.	Solve $\frac{dy}{dx}=y\tan x-2\sin x$.	[4]	CO2
3.	Find an integrating factor of $\left(y+\frac{y^3}{3}+\frac{x^2}{2}\right)dx+\left(\frac{x}{4}+\frac{xy^2}{4}\right)dy=0$.	[4]	CO2
4.	Find the nature of solution of the differential equation $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$	[4]	CO1
5.	Let $f(D)y=\sin(ax)$ be a linear n^{th}order differential equation then show that the particular integral $\frac{1}{f(D^2)}\sin(ax)=\frac{1}{f(-a^2)}\sin(ax)$ provided $f(-a^2)\neq 0$.	[4]	CO3

SECTION B
(All questions are compulsory, Q10 has internal choice)

6.	Find the equilibrium solutions of the autonomous equation $y'=(y^5-4y^3+y^2-4)$ and hence determine their stability.	[08]	CO5
7.	Develop a model based on the following assumptions and hence determine the population size with $x(0)=x_0$. i. Assume that the populations are sufficiently large so that we can ignore random differences between individuals ii. Assume that births and deaths are continuous in time iii. Assume that per-capita birth and death rates are constant in time iv. Ignore immigration and emigration	[08]	CO4

8.	Solve $\frac{dy}{dx}+\frac{y}{\left(1-x^2\right)^{\frac{3}{2}}}=\frac{x+\sqrt{1-x^2}}{\left(1-x^2\right)^2}$	**[08]**	**CO2**
9.	Explain some classifications of mathematical models.	**[08]**	**CO4**
10.	Solve $\frac{d^2 y}{d x^2}-4\frac{dy}{dx}+4y=3x^2 e^{2x}\sin(2x)$ **OR** Solve $\left(D^4-1\right)y=x\,sinx$	**[08]**	**CO3**
SECTION C **(Q11 is compulsory and Q12A, Q12B have internal choice**			
11. A	Consider the following system: $x^{'}=x-2y+2z\,;\,y^{'}=-4x+3y+2z\,;\,z^{'}=4x-2y-z$ Determine the stability of the equilibrium point the origin.	**[10]**	**CO5**
11.B	Solve the Cauchy-Euler equation $x^3\frac{d^3 y}{d x^3}+2x^2\frac{d^2 y}{d x^2}+2y=10\left(x+\frac{1}{x}\right)$	**[10]**	**CO3**
12. A	Consider the differential equation $y=2px+p^4 x^2$ where $p=\frac{dy}{dx}$ (i) Find a one-parameter family of solutions (ii) Find an extra solution (if exists) that is not a member of the one-parameter family found in part (i). **OR** Find the general solution and singular solution (if exists) of the differential equation $p^2 x(x-2)+p(2y-2xy-x+2)+y^2+y=0$where $p=\frac{dy}{dx}$.	**[10]**	**CO2**
12.B	Solve $y^{''}-2y^{'}-3y=4x-5+6xe^{2x}$ using method of undetermined coefficients. **OR** Solve $4y^{''}+36y=cosec(3x)$ using variation of parameters method.	**[10]**	**CO3**

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