Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: B. Tech. ASE **Semester : VII**
Course Name : Computational Fluid Dynamics **Time : 03 hrs.**
Course Code : GNEG 401 **Max. Marks: 100**
Nos. of page(s) : 03
Instructions: Assume any missing data appropriately.

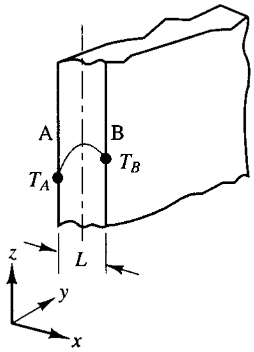
SECTION A

S. No.		Marks	CO
Q 1	List the various physical boundary conditions encountered in a non-isothermal fluid flow.	4	CO1
Q 2	Discuss the advantages and disadvantages of unstructured grids over structured grids.	4	CO3
Q 3	Classify the following equations as hyperbolic, parabolic or elliptic. a. $\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$ b. $\frac{\partial u}{\partial t}+c\frac{\partial u}{\partial x}=0$ c. $\frac{\partial^2 \varnothing}{\partial x^2}+\frac{\partial^2 \varnothing}{\partial y^2}=0$ d. $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$	4	CO1
Q 4	Consider the function $\phi(x, y) = e^x + e^y$. a. Calculate the values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at a point $(x,y) = (1,1)$ using first order forward difference, with $\Delta x = \Delta y = 0.1$. b. Calculate the values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at a point (x,y) = (1,1) using second order central difference, with $\Delta x = \Delta y = 0.1$.	4	CO3
Q 5	Formulate any two approximations for the evaluation surface integral of fluxes over the east face of a two-dimensional control volume.	4	CO2

SECTION B

Q 6	Define the CDS interpolation scheme for the evaluation of fluxes at face centre	10	CO2

	using the nodal values on a structured finite volume grid. Find the order of accuracy of this scheme and discuss its advantages and disadvantages.		
Q 7	Illustrate the strong and weak forms of the weighted residual formulation for finite element discretization. Justify that a proper choice of *weight function* makes the weighted residual formulation equivalent to Finite difference or Finite Volume Methods. ***OR*** Define shape functions as used in Finite Element Method. Deduce shape functions for a one-dimensional quadratic element for the value of a function at any location in the domain in terms of nodal values.	**10**	**CO2**
Q 8	Illuminate the need of a body fitted coordinate system for the solution of governing flow equations using finite difference method. Explain thus, the philosophy of elliptic grid generation around an airfoil.	**10**	**CO3**
Q 9	Discuss the explicit McCormack time marching algorithm for the solution of transient Euler equations in 2-dimensions.	**10**	**CO2**
SECTION-C			
Q 10	Consider a two-dimensional square plate ABCD with edges AB and CD maintained at temperatures of 200 *ºK* and 100 *ºK* respectively. The other two edges DA and BC are also maintained at temperatures of 200 *ºK*, except at the corners C and D. Find the steady state temperatures of at least 9 locations on the plate. Take *AB=BC=CD=DA*= 4 cm. Use pure Gauss-Seidel relaxation scheme for at least 4 iterations. The two-dimensional steady state heat conduction is governed by $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$ ***OR*** Consider a large flat plate of thickness L = 2 cm with constant thermal conductivity *k* = 0.5 W/m.K and uniform heat generation *q* =1000 kW/m^3. The opposite faces A and B, as shown in figure below at maintained at temperatures of 100 ºC and 200 ºC respectively. Assuming the heat conduction to be one-dimensional, estimate the steady state temperature distribution in the plate.	**20**	**CO5**

	The governing equation can be assumed as $\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+q=0$		
Q 11	Derive the *modified equation* that emanates from the first order forward in time and backward in space discretization of the first order wave equation given below. $$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0$$ Discuss the nature of dominating error for the above discretization and suggest means to minimize them.	**20**	**CO4**

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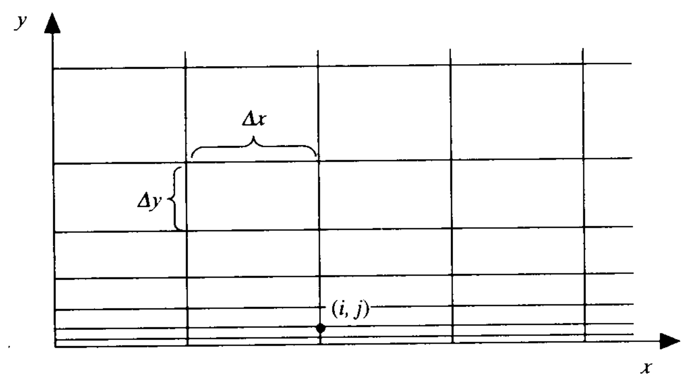
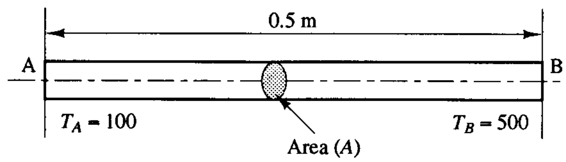
SECTION A

S. No.		Marks	CO
Q 1	Sketch the various models of fluid flow used for derivation of governing equations. Write down the forms of equations that emanate from these models on applications conservation laws.	4	CO1
Q 2	Consider the viscous flow of air over a flat plate. At a given station in the flow direction, the variation of the flow velocity, u, in the direction perpendicular to the plate (the y direction) is given at discrete grid points equally spaced in y direction with Δy = 2.54 mm. **y (mm)** — **u (m/s)** 0 — 0 2.54 — 45.72 5.08 — 87.41 7.62 — 125.0 Imagine that the values of $\boldsymbol{u}$ listed above are discrete values at discrete grid points located at $\boldsymbol{y}$ = 0, 2.54, 5.08 and 7.62 mm the same nature as would be obtained from a numerical finite difference solution of the flow field. For viscosity coefficient, μ =1.7895 x 10^{-5} kg/m-s, using these discrete values; Calculate the shear stress at the wall $\boldsymbol{\tau_w}$ three different ways, namely: a. Using a first order one sided difference b. Using the second order one sided difference	4	CO3
Q 3	Define *numerical diffusion and dispersion.* Discuss the effect of numerical diffusion and dispersion on the solution of the one-dimensional scalar wave equation using the explicit Forward in Time and Backward in Space (FTBS) scheme. Suggest methods	4	CO4

	to alleviate the diffusive error.		
Q 4	Write formulae for approximation of surface integrals of fluxes over a control volume face using following methods. a. Mid-point method b. Trapezoidal Method c. Simpson's Method	**4**	**CO3**
Q 5	Elucidate the need of grid and equation transformation for the solution flow over complex geometries using finite difference method.	**4**	**CO3**
	SECTION B		
Q 6	Consider the numerical solution of steady viscous flow over a flat plate using a finite difference scheme. To calculate the details of this flow near the surface, very fine mesh, stretched in transverse direction is required as shown in figure below.  The solution requires an equispaced Cartesian grid in computational plane (ξ, η) which can be obtained through following direct transformations $\xi = x$ $\eta = \ln(1 + y)$ If the continuity equation for above flow in physical plane (*x*, *y*) is $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$, find the continuity equation that is required to be solved in the computational plane.	**10**	**CO3**
Q 7	Consider the 2-dimensional transient heat conduction equation given below. The Crank-Nicolson discretization of the equation results in a pentadiagonal system of	**10**	**CO2**

	equations. Demonstrate an algorithm to solve the system of equations iteratively. $$\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$		
Q 8	Discuss the solution of Laplace equation in 2-dimensions using an explicit five point Gauss-Seidel Scheme point iterative scheme. Suggest modifications that can accelerate this scheme.	**10**	**CO2**
Q 9	Elaborate the following interpolation schemes for the evaluation of flux values at the center of a control volume face using the flux values at computational nodes for a structured finite volume grid. a. Central Differencing Scheme b. Hybrid Scheme Discuss the advantages, disadvantages and the order of accuracy of each of these schemes. ***OR*** Define the UPWIND interpolation scheme for the evaluation of fluxes at face centre using the nodal values on a structured finite volume grid. Find an expression for the artificial diffusivity introduced by this scheme.	**10**	**CO2**
	SECTION-C		
Q 10	Deduce the *modified equation* for the solution of the first order wave equation using Lax Method given by $$\frac{u_j^{n+1}-(u_{j+1}^n+u_{j-1}^n)/2}{\Delta t}+c\frac{u_{j+1}^n-u_{j-1}^n}{2\,\Delta x}=0$$ Hence, discuss the effect of the dominating error on the solution obtained.	**20**	**CO4**
Q 11	Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of 100 °C and 500 °C respectively. The one-dimensional problem sketched in Figure below, is governed by $$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right)=0$$	**20**	**CO5**

Calculate the steady state temperature distribution in the rod. Thermal conductivity k equals 1000 W/m/K; cross-sectional area A is 10 x 10^{-3} m^2. Use at least 5 control volumes with appropriate interpolation scheme.



OR

Consider square plate PQRS whose edges PQ, QR and SP are maintained at temperature of 300 ºK whereas the edge RS is maintained at 100 ºK. Find the steady state temperatures of at least 9 locations on the plate. Take $PQ=QR=RS=SP=$ 4 cm. Use a point iterative relaxation scheme for at least 4 iterations with an over-relaxation factor of 1.2.

The two-dimensional steady state heat conduction is governed by

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$