Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Course: MATH 1030 Calculus **Semester: I**

Programme: B. Sc. (H) Mathematics

Time: 03 hrs. **Max. Marks: 100**

Instructions:

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A

(Attempt all questions)

1.	If $y = e^{ax}\sin bx$, prove that $y_2 - 2ay_1 + (a^2+b^2)y = 0$.	**[4]**	**CO1**
2.	Evaluate $\int \sin^4 x\,dx$.	**[4]**	**CO2**
3.	Find the eccentricity of the given conic section $17x^2 - 12xy + 8y^2 + 46x - 28y + 17 = 0$.	**[4]**	**CO3**
4.	Find the coordinate of the centre of the conic $11x^2 + 24xy + 9y^2 - 130ax - 60ay + 116a^2 = 0$.	**[4]**	**CO3**
5.	Find a such that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} + 2\hat{j} - 3\hat{k},$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar.	**[4]**	**CO4**

SECTION B

(Q6-Q8 are compulsory and Q9 has internal choice)

6.	If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$.	**[10]**	**CO1**
7.	Evaluate $\lim_{x\to 0}\dfrac{x^x - x}{x - 1 - \log x}$.	**[10]**	**CO1**
8.	Find the area of the tangent cut off from the parabola $x^2 = 8y$ by the lines $x - 2y + 8 = 0$.	**[10]**	**CO2**
9.	Find the perimeter of the loop of the curve $3ay^2 = x(x-a)^2$.	**[10]**	**CO2**

	OR Find the volume formed by the revolution of the loop of the curve $y^2(a+x)=x^2(3a-x)$, about the x-axis.		
	SECTION C **(Q10 is compulsory and Q11 has internal choice)**		
10.A	Find the coordinate of focus and vertex of given conic section $9x^2-24xy+16y^2-18x-101y+19=0.$	**[10]**	**CO3**
10.B	Given $\vec{r}=t^m A+t^n B$, where A, B are constant vectors, show that, if $\vec{r}$ and d^2r/dt^2 are parallel vectors, then $m+n=1$, unless $m=n$.	**[10]**	**CO4**
11.A	Trace the conic $5x^2+4xy+8y^2-12x-12y=0$. **OR** Show that the locus of the pole of given straight line with respect to a series of confocal conics is a straight line.	**[10]**	**CO3**
11.B	Given $R(t)=3t^2\hat{i}+t\hat{j}-t^3\hat{k}$, evaluate $\int_0^1 \left(R\times d^2R/dt^2\right)dt$. **OR** If $F=3xy\hat{i}-y^2\hat{j}$, evaluate $\int_C F.dr$, where C is the curve in the xy- plane $y=2x^2$ from (0,0) to (1,2).	**[10]**	**CO4**

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Section A

(Attempt all questions)

1.	If $y=\sin(\sin x)$, prove that $y_2+\tan x\, y_1+y\cos^2 x=0$.	**[4]**	**CO1**
2.	Evaluate $\int cos^4 x\, dx$.	**[4]**	**CO2**
3.	Find the eccentricity of the given conic section $34x^2-24xy+16y^2+92x-56y+34=0$.	**[4]**	**CO3**
4.	Find the coordinate of the centre of the conic $22x^2+48xy+18y^2-260ax-120ay+232a^2=0$.	**[4]**	**CO3**
5.	Show that the points $-6\hat{i}+3\hat{j}+2\hat{k}, 3\hat{i}-2\hat{j}+4\hat{k}, 5\hat{i}+7\hat{j}+3\hat{k}$ and $-13\hat{i}+17\hat{j}-\hat{k}$ are coplanar.	**[4]**	**CO4**

SECTION B

(Q6-Q8 are compulsory and Q9 has internal choice)

6.	If $V_n=\frac{d^n}{dx^n}(x^n\log x)$, show that $V_n=nV_{n-1}+n-1!$.	**[10]**	**CO1**
7.	Evaluate $\lim_{x\to 0}\frac{(1+x)^x-e}{x}$.	**[10]**	**CO1**
8.	Find the area common to the parabola $y^2=ax$ and the circle $x^2+y^2=4ax$.	**[10]**	**CO2**
9.	Find the length of the arc of the parabola $x^2=4ay$ measured from the vertex to one extremity of the latus-rectum.	**[10]**	**CO2**

	OR Find the volume of a sphere of radius a.		
SECTION C **(Q10 is compulsory and Q11 has internal choice)**			
10.A	Find the coordinate of focus and vertex of given conic section $x^2 - 4xy + 4y^2 - 12x - 6y - 39 = 0$.	**[10]**	**CO3**
10.B	If $R(t) = \begin{cases} 2i - j + 2k & \text{when } t = 1 \\ 3i - 2j + 4k & \text{when } t = 2 \end{cases}$, show that $\int_1^2 (R.\frac{dR}{dt})dt = 10$.	**[10]**	**CO4**
11.A	Trace the conic $5x^2 + 4xy + 8y^2 - 12x - 12y = 0$. **OR** Show that the locus of the pole of given straight line with respect to a series of confocal conics is a straight line.	**[10]**	**CO3**
11.B	Given $R(t) = (5t^2 - 3t)\hat{i} + 6t^3\hat{j} - 7t\hat{k}$, evaluate $\int_2^4 (F(t))\, dt$. **OR** A vector field is given by $F = \sin y\hat{i} + x(1 + \cos y)\hat{j}$, evaluate the line integral over a circular path given by $x^2 + y^2 = a^2, z = 0$.	**[10]**	**CO4**