Name:	ame: prolment No:		
Enrolment No:			
	UNIVERSITY OF PETROLEUM AND ENERGY STUDIES		
Course	End Semester Examination, December 2018 : Mathematical Physics-I (PHYS 1011) Semester: I		
	: Mathematical Physics-I (PHYS 1011) Semester: I mme: BSc Physics (H) Time: 03 hrs	i -	
0	Iarks: 100	•	
	r of pages: 03		
Instruc	• •		
	SECTION A		
	All questions are compulsory.		
SN	Statement of Question	Marks	CO
Q1	Define scalar and vector fields. Plot the following vector field (no need of graph		GOA
	paper): $\vec{A}(u, v) = -2v\hat{v} + v\hat{v}$	4	CO3
Q2	$\vec{A}(x,y) = -2x\hat{\imath} + y\hat{\jmath}$ What do you mean by directional derivative of a scalar field? Estimate the		
Q^2	directional derivative of the following scalar function		G 0 0
	$\varphi = x^2 yz + 4xz^2$	4	CO3
	at (1,-2,-1).		
Q3	Define Dirac Delta function and state its properties.	4	CO2
Q4	Prove that the vector field $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2x)\hat{\imath}$	4	CO3
	$(2xz + 2z)\hat{k}$ is solenoidal.	-	005
Q5	Using Lagrange Multiplier's method, compute the maxima and minima of the		
	function $f(x, y, z) = x^2 - y^2$	4	CO2
	On the surface $f(x, y, z) = x - y$	-	02
	$x^2 + 2y^2 + 3z^2 = 1$ SECTION B		
	Questions 6-8 are compulsory. There is internal choice for question number	9.	
Q6	Using Wronskian's method of variation of parameters, find the complete solution of		
	the following differential equation:	10	CO2
	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$		
Q7	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$ If \vec{A} is a vector field given as $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ then prove		
χ '	$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla} (\vec{\nabla} \cdot \vec{A})$	10	CO3
		10	005
Q8	where ∇ and ∇^2 are DEL and Laplacian operators, respectively. Prove if the following differential equation		
V 0	$(2xy + y - \tan y dx + (x^2 - x \tan^2 y + \sec^2 y) dy) = 0$	10	CO1
	is exact. Find the solution of the differential equation.	-	
Q9	State Gauss's Divergence theorem for a vector field. Using Gauss's divergence	10	CO4
	theorem, evaluate	10	

			1
	$\int \vec{F} \cdot \hat{n} ds$		
	on a surface S of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. The		
	vector field is given as $\vec{F} = 4xz \hat{\imath} - y^2 \hat{\jmath} + yz \hat{k}$ and \hat{n} is the unit vector normal to		
	the surface S. OR		
Q9	State Stoke's theorem for a vector field. Evaluate $\oint \vec{F} \cdot \vec{dr}$ on a closed surface C in		
C	the xy plane, $x = 2 \cos t$, $y = 3 \sin t$ from $t = 0$ to $t = 2\pi$. The vector field \vec{F} is		
	given as	10	CO4
	$\vec{F} = (x - 3y)\hat{\imath} + (y - 2x)\hat{\jmath}$	10	004
	[Note: No need to use Stoke's theorem for evaluating $\oint \vec{F} \cdot \vec{dr}$. Evaluate it as a line integral]		
	SECTION-C		
	Q10 is compulsory. There is an internal choice for Q11.	1	
Q10	a) If $\vec{R}(x) = (x - x^2)\hat{i} + 2x^3\hat{j} - 3\hat{k}$ then calculate $\int_1^2 \vec{R}(x)dx$		
	b) When do we call a vector irrotational? Find constants a, b, c so that the vector		
	$\vec{V} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$		
	is irrotational.c) The differential equation corresponding to the rate of change of current with		CO4+
	time in a circuit containing an inductance L and a resistance R in series acted	5+5+10	CO3+
	on by an electromotive force $E \sin \omega t$ is		CO1
	$iR + L\frac{di}{dt} = E\sin\omega t$		
	ut		
	Find the value of current at any time t , if initially there is no current in the circuit. Draw the current Vs phase (ωt) graph and comment on its behavior.		
Q11	a) Find the area of a triangle with vertices at (3,-1,2), (1,-1,-3) and (4,-3,1).		
	b) The temperature of points in the space is given by		
	$T(x, y, z) = x^2 + y^2 - z$		
	A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move?		
	c) Evaluate $\int \vec{F} \cdot \vec{dS}$ where $\vec{F} = 4x \hat{i} - 2y^2 \hat{j} + z^2 \hat{k}$ and S is the surface		
	bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ (see the figure below).		
	$Z^{\uparrow} = 3$		CO3+
		5+5+10	CO3+
	and the second se		CO4
	- contraction - contraction		
	$x^2 + x^2 = 4$		
	X $z = 0$ Y		
<u> </u>		1	1

	OR		
Q11	a) Find the divergence and curl of		
	$\vec{v} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 - y^2z)\hat{k}$		
	b) Find the volume of parallelepiped if $\vec{a} = -3\hat{\imath} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -3\hat{\imath} + 7\hat{j} - 3\hat{k}$		
	$3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the co-terminus (originating from the same vertex)		CO3+
	edges of the parallelepiped.	5+5+10	CO3+
	c) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + \hat{\imath}$		CO4
	$(2xz - y)\hat{j} + z\hat{k}$ along		
	I. Straight line from $(0, 0, 0)$ to $(2, 1, 3)$		
	II. The curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.		

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SN	Statement of Question	Marks	CO
Q1	Define partial derivative of a function $f(x, y) = c$ w.r.t x with the help of limits. If	1 1141 N 3	
Υ ¹	the function is given as		
	$f(x, y) = \tan^{-1}(y/x)$	4	CO2
	$f(x, y) = \tan^{-1}(y/x)$ calculate $\frac{\partial^2 f}{\partial x^2}$ and prove that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.		
02	$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial x^2}$		
Q2	Write the statement of Gauss's Divergence theorem and explain its physical meaning.	4	CO4
Q3	What are the necessary and sufficient conditions for the differential equation		
X ²	M(x,y)dx + N(x,y)dy = 0 to be exact? Prove if the following differential		001
	equation is exact:	4	CO1
	$(ye^{x} + 2xy^{2})dx - e^{x}dy = 0$ Using Lagrange Multiplier's method, estimate the maxima of the function		
Q4			
	f(x, y, z) = xy	4	CO2
	On the surface $2x^2 + x^2 = 6$		
Q5	$3x^2 + y^2 = 6$		001
X 2	Find the general solution of $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0.$	4	CO1
	SECTION B		
	Questions 6-8 are compulsory. There is an internal choice for Q9		I
Q6	Using the method of variation of parameters, find out the complete solution of the		
	following differential equation $d^2 y = dy = 1$	10	CO1
	$\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = \frac{1}{1 + e^{-x}}$		
Q7	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1 + e^{-x}}$ Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, and \vec{a} is a constant vector ($\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$). Prove that		
ر .	$\int d\vec{a} \times \vec{r}$	10	CO3
	$\vec{\nabla} \cdot \left(\frac{\vec{a} \times \vec{r}}{r^n}\right) = 0$	Ĩ	
Q8	Explain the physical significance of directional derivative of a scalar function. Find		
•	the directional derivative of \vec{v}^2 , where	10	CO3
	$\vec{v} = xy^2\hat{\imath} + zy^2\hat{\jmath} + xz^2\hat{k}$		

$\begin{array}{c c} \cos p \operatorname{all} \ \operatorname{cond} \ \operatorname{min} \ \operatorname{cond} \ \operatorname{min} \ \operatorname{cond} \ \operatorname{min} \ \operatorname{cond} \ \operatorname{min} \ \operatorname{min} \ \operatorname{min} \ \operatorname{cond} \ \operatorname{min} \ m$		at the point (2, 0, 3) in the direction $6\hat{i} + 4\hat{j} + 2\hat{k}$.		
$\begin{array}{c c} \mbox{compute the line integral } \oint \vec{A} \cdot d\vec{r} \mbox{ from } (0, 0, 0) \mbox{ to } (1, 1, 1) \mbox{ along the curve } C \mbox{ given } as x = t, y = t^2, z = t^3. \\ \hline & OR \\ \hline \\ $	Q9	Explain Stokes' theorem for a vector field. If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$,		
ORIfQ9If $\varphi = 2xyz^2$, $\vec{F} = xy\hat{t} - z\hat{j} + x^2\hat{k}$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$, evaluate the following line integrals: a) $\oint \phi \vec{P} \times d\vec{r}$ where $\vec{r} = x\hat{t} + y\hat{j} + z\hat{k}$ 10Q10a) If $\vec{A} = 2yz\hat{t} - x^2y\hat{j} + xz^2\hat{k}$ and $\varphi = 2x^2yz^3$, calculate $(\vec{A} \cdot \vec{\nabla})\varphi$ b) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal, where $\vec{r} = x\hat{t} + y\hat{j} + z\hat{k}$ 5+5+10Q10a) If $\vec{A} = 2yz\hat{t} - x^2y\hat{j} + xz^2\hat{k}$ and $\varphi = 2x^2yz^3$, calculate $(\vec{A} \cdot \vec{\nabla})\varphi$ b) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal, where $\vec{r} = x\hat{t} + y\hat{j} + z\hat{k}$ 5+5+10Q11a) Suppose that a cylindrical can is designed to have a radius of 1 inch and a height of 5 inch, but that the radius and height are off by the amounts $dr = 0.03$ inch and $dh = -0.1$ inch. Estimate the resulting absolute change in the volume of the can. b) Show that the differential equation $(xy^2 - e^{\frac{1}{x^3}}) dx - x^2y dy = 0$ is not exact. Find the integrating factor for solving this equation. c) The heat flow vector $\vec{H} = k\vec{T}$, where T is the temperature and k is the thermal conductivity. Show that where $T = 50 \sin \frac{\pi x}{2} \cosh \frac{\pi y}{2}$ then $\vec{\nabla} \cdot \vec{H} = 0$.C02 C02 C03 C03Q11a) The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ Show that5+5+10		compute the line integral $\oint \vec{A} \cdot \vec{dr}$ from (0, 0, 0) to (1, 1, 1) along the curve C given	10	CO4
Q9If $\varphi = 2xyz^2$, $\vec{F} = xy^2 - zj + x^2k$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$, evaluate the following line integrals: a) $\oint \varphi dr$ b) $\oint \vec{F} \times d\vec{r}$ where $\vec{r} = x\hat{t} + y\hat{f} + z\hat{k}$ 10COSECTION-C Q10 is compulsory. There is an internal choice for Q11Q10a) If $\vec{A} = 2yz\hat{t} - x^2y\hat{f} + xz^2\hat{k}$ and $\varphi = 2x^2yz^3$, calculate $(\vec{A} \cdot \vec{\nabla})\varphi$ b) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal, where $\vec{r} = x\hat{t} + y\hat{f} + z\hat{k}$ c. Verify the divergence theorem $\oint \vec{A} \cdot d\vec{L} = \iiint \vec{\nabla} \cdot \vec{A} dv$ 5+5+10CO3 CO3 CO3 CO3Q11a) Suppose that a cylindrical can is designed to have a radius of 1 inch and a height of 5 inch, but that the radius and height are off by the amounts $dr = 0.03$ inch and $dh = -0.1$ inch. Estimate the resulting absolute change in the volume of the can. b) Show that the differential equation $(xy^2 - e^{\frac{1}{x^2}}) dx - x^2ydy = 0$ is not exact. Find the integrating factor for solving this equation. c. The heat flow vector $\vec{H} = k\vec{T}$, where T is the temperature and k is the thermal conductivity. Show that where $T = 50 \sin \frac{\pi x}{2} \cosh \frac{\pi y}{2}$ 5+5+10CO2 CO2 CO2 CO3Q11a) The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ Show that5+5+10CO2 CO2 CO3				
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Q10a) If $\vec{A} = 2yz i - x^2y j + xz^2 k$ and $\varphi = 2x^2yz^3$, calculate $(\vec{A} \cdot \vec{\nabla}) \varphi$ b) Find the value of n for which the vector $r^n \vec{r}$ is solenoidal, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ c) Verify the divergence theorem $\oint \vec{A} \cdot d\vec{S} = \iiint \vec{\nabla} \cdot \vec{A} dv$ 5+5+10CO3Q11a) Suppose that a cylindrical can is designed to have a radius of 1 inch and a height of 5 inch, but that the radius and height are off by the amounts $dr = 0.03$ inch and $dh = -0.1$ inch. Estimate the resulting absolute change in the volume of the can.5+5+10CO2(C01)(xy^2 - ex^{\frac{1}{x^3}}) dx - x^2y dy = 0is not exact. Find the integrating factor for solving this equation.5+5+10CO2(C02)(xy^2 - ex^{\frac{1}{x^3}}) dx - x^2y dy = 0is not exact. Find the integrating factor for solving this equation.5+5+10CO2(C03)(xy^2 - ex^{\frac{1}{x^3}}) dx - x^2y dy = 0is not exact. Find the integrating factor for solving this equation.5+5+10CO2(C03)(C04)(C04)(C05)(C05)(C05)(C04)(C05)(C06)(C06)(C06)(C05)(C06)(C07)(C07)(C07)(C07)(C08)(C08)(C08)(C08)(C01)(C07)(C07)(C07)(C07)(C02)(C08)(C08)(C08)(C08)(C03)(C08)(C08)(C08)(C08)(C01)(C08)(C08)(C08)(C08)(C01)(C08)(C08)(C08)(C08)(C01)(C08)(C08)(C08)(C01)(C08)(C08)(C08				
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\cdot 1 \cdot \cdot \cdot \cdot \cdot \cdot				CO3
\cdot 1 \cdot \cdot \cdot \cdot \cdot \cdot		$dR = \left(\frac{R}{R_1}\right) dR_1 + \left(\frac{R}{R_2}\right) dR_2$		
		b) Prove that the differential equation (R_2)		

	$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$
is homogene	bus in x and y. Estimate the integrating factor to solve this equation.
c) Prove that	t the vector field given by
	$\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$
is irrotati	onal. Find its scalar potential.