Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2018

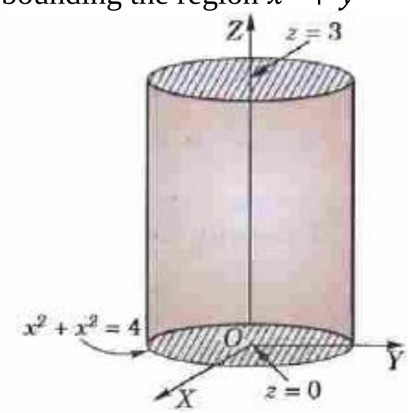
Course: Mathematical Physics-I (PHYS 1011) **Semester: I**
Programme: BSc Physics (H) **Time: 03 hrs.**
Max. Marks: 100
Number of pages: 03
Instructions:

SECTION A
All questions are compulsory.

SN	Statement of Question	Marks	CO
Q1	Define scalar and vector fields. Plot the following vector field (no need of graph paper): $$\vec{A}(x,y) = -2x\hat{\imath} + y\hat{\jmath}$$	**4**	**CO3**
Q2	What do you mean by directional derivative of a scalar field? Estimate the directional derivative of the following scalar function $$\varphi = x^2yz + 4xz^2$$ at (1,-2,-1).	**4**	**CO3**
Q3	Define Dirac Delta function and state its properties.	**4**	**CO2**
Q4	Prove that the vector field $\vec{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is solenoidal.	**4**	**CO3**
Q5	Using Lagrange Multiplier's method, compute the maxima and minima of the function $$f(x,y,z) = x^2 - y^2$$ On the surface $$x^2 + 2y^2 + 3z^2 = 1$$	**4**	**CO2**

SECTION B
Questions 6-8 are compulsory. There is internal choice for question number 9.

SN	Statement of Question	Marks	CO
Q6	Using Wronskian's method of variation of parameters, find the complete solution of the following differential equation: $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x$$	**10**	**CO2**
Q7	If $\vec{A}$ is a vector field given as $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ then prove $$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2\vec{A} + \vec{\nabla}(\vec{\nabla}\cdot\vec{A})$$ where $\vec{\nabla}$ and ∇^2 are DEL and Laplacian operators, respectively.	**10**	**CO3**
Q8	Prove if the following differential equation $$(2xy + y - \tan y\, dx + (x^2 - x\tan^2 y + \sec^2 y)dy) = 0$$ is exact. Find the solution of the differential equation.	**10**	**CO1**
Q9	State Gauss's Divergence theorem for a vector field. Using Gauss's divergence theorem, evaluate	**10**	**CO4**

	$$\iint \vec{F} \cdot \hat{n}\, ds$$ on a surface S of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. The vector field is given as $\vec{F} = 4xz\,\hat{\imath} - y^2\,\hat{\jmath} + yz\,\hat{k}$ and $\hat{n}$ is the unit vector normal to the surface S. **OR**		
Q9	State Stoke's theorem for a vector field. Evaluate $\oint \vec{F} \cdot \overrightarrow{dr}$ on a closed surface C in the xy plane, $x = 2\cos t, y = 3\sin t$ from $t = 0$ to $t = 2\pi$.The vector field $\vec{F}$ is given as $$\vec{F} = (x - 3y)\hat{\imath} + (y - 2x)\hat{\jmath}$$ [**Note**: No need to use Stoke's theorem for evaluating $\oint \vec{F} \cdot \overrightarrow{dr}$. Evaluate it as a line integral]	**10**	**CO4**
SECTION-C **Q10 is compulsory. There is an internal choice for Q11.**			
Q10	a) If $\vec{R}(x) = (x - x^2)\hat{\imath} + 2x^3\hat{\jmath} - 3\hat{k}$ then calculate $\int_1^2 \vec{R}\,(x)dx$ b) When do we call a vector irrotational? Find constants a, b, c so that the vector $$\vec{V} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$$ is irrotational. c) The differential equation corresponding to the rate of change of current with time in a circuit containing an inductance L and a resistance R in series acted on by an electromotive force $E \sin \omega t$ is $$iR + L\frac{di}{dt} = E \sin \omega t$$ Find the value of current at any time t, if initially there is no current in the circuit. Draw the current Vs phase (ωt) graph and comment on its behavior.	**5+5+10**	**CO4+** **CO3+** **CO1**
Q11	a) Find the area of a triangle with vertices at (3,-1,2), (1,-1,-3) and (4,-3,1). b) The temperature of points in the space is given by $$T(x, y, z) = x^2 + y^2 - z$$ A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move? c) Evaluate $\int \vec{F} \cdot \overrightarrow{dS}$ where $\vec{F} = 4x\,\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$ (see the figure below).  	**5+5+10**	**CO3+** **CO3+** **CO4**

	OR		
Q11	a) Find the divergence and curl of $$\vec{v} = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 - y^2z)\hat{k}$$ b) Find the volume of parallelepiped if $\vec{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}, \vec{b} = -3\hat{\imath} + 7\hat{\jmath} - 3\hat{k}, \vec{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$ are the co-terminus (originating from the same vertex) edges of the parallelepiped. c) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along I. Straight line from (0, 0, 0) to (2, 1, 3) II. The curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.	**5+5+10**	**CO3+ CO3+ CO4**

Set-II

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Instructions:

SECTION A

All questions are compulsory.

SN	**Statement of Question**	**Marks**	**CO**
Q1	Define partial derivative of a function $f(x,y) = c$ w.r.t x with the help of limits. If the function is given as $$f(x,y) = \tan^{-1}(y/x)$$ calculate $\frac{\partial^2 f}{\partial x^2}$ and prove that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.	**4**	**CO2**
Q2	Write the statement of Gauss's Divergence theorem and explain its physical meaning.	**4**	**CO4**
Q3	What are the necessary and sufficient conditions for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact? Prove if the following differential equation is exact: $$(ye^x + 2xy^2)dx - e^x dy = 0$$	**4**	**CO1**
Q4	Using Lagrange Multiplier's method, estimate the maxima of the function $$f(x,y,z) = xy$$ On the surface $$3x^2 + y^2 = 6$$	**4**	**CO2**
Q5	Find the general solution of $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$.	**4**	**CO1**
	SECTION B **Questions 6-8 are compulsory. There is an internal choice for Q9**		
Q6	Using the method of variation of parameters, find out the complete solution of the following differential equation $$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{-x}}$$	**10**	**CO1**
Q7	Let $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, and $\vec{a}$ is a constant vector ($\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$). Prove that $$\vec{\nabla} \cdot \left(\frac{\vec{a} \times \vec{r}}{r^n}\right) = 0$$	**10**	**CO3**
Q8	Explain the physical significance of directional derivative of a scalar function. Find the directional derivative of $\vec{v}^2$, where $$\vec{v} = xy^2\hat{\imath} + zy^2\hat{\jmath} + xz^2\hat{k}$$	**10**	**CO3**

	at the point (2, 0, 3) in the direction $6\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$.		
Q9	Explain Stokes' theorem for a vector field. If $\vec{A} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$, compute the line integral $\oint \vec{A} \cdot \overrightarrow{dr}$ from (0, 0, 0) to (1, 1, 1) along the curve C given as $x = t, y = t^2, z = t^3$.	**10**	**CO4**
	OR		
Q9	If $\varphi = 2xyz^2$, $\vec{F} = xy\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$ and C is the curve $x = t^2, y = 2t, z = t^3$ from $t = 0$ to $t = 1$, evaluate the following line integrals: a) $\oint \varphi dr$ b) $\oint \vec{F} \times \overrightarrow{dr}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$	**10**	**CO4**
SECTION-C **Q10 is compulsory. There is an internal choice for Q11**			
Q10	a) If $\vec{A} = 2yz\,\hat{\imath} - x^2y\,\hat{\jmath} + xz^2\hat{k}$ and $\varphi = 2x^2yz^3$, calculate $(\vec{A} \cdot \vec{\nabla})\varphi$ b) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ c) Verify the divergence theorem $$\oint \vec{A} \cdot \overrightarrow{dS} = \iiint \vec{\nabla} \cdot \vec{A}\, dv$$ For $\vec{A} = xy^2\hat{\imath} + y^3\hat{\jmath} + y^2z\hat{k}$ and S is surface of the cuboid defined by $0 < x < 1, 0 < y < 1, 0 < z < 1$	**5+5+10**	**CO3+ CO3+ CO4**
Q11	a) Suppose that a cylindrical can is designed to have a radius of 1 inch and a height of 5 inch, but that the radius and height are off by the amounts $dr = 0.03$ inch and $dh = -0.1$ inch. Estimate the resulting absolute change in the volume of the can. b) Show that the differential equation $$\left(xy^2 - e^{\frac{1}{x^3}}\right)dx - x^2y\,dy = 0$$ is not exact. Find the integrating factor for solving this equation. c) The heat flow vector $\vec{H} = k\overrightarrow{\nabla T}$, where T is the temperature and k is the thermal conductivity. Show that where $$T = 50 \sin\frac{\pi x}{2} \cosh\frac{\pi y}{2}$$ then $\vec{\nabla} \cdot \vec{H} = 0$.	**5+5+10**	**CO2+ CO2+ CO3**
	OR		
Q11	a) The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel can be calculated from the formula: $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$ Show that $$dR = \left(\frac{R}{R_1}\right)^2 dR_1 + \left(\frac{R}{R_2}\right)^2 dR_2$$ b) Prove that the differential equation	**5+5+10**	**CO2+ CO2+ CO3**

	$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$ is homogeneous in x and y. Estimate the integrating factor to solve this equation. c) Prove that the vector field given by $$\vec{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$$ is irrotational. Find its scalar potential.		