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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme: M.Tech. (PIPELINE) **Semester – I**
Course Name: Numerical Methods In Engineering **Max. Marks : 100**
Course Code: CHPL-7003 **Duration : 3 Hrs**
No. of page/s: 02

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

SECTION A
(Attempt all questions)

1.	Obtain the polynomial x^3-5x+1 in factorial notation.	**[4]**	**CO1**
2.	Consider the symmetric matrix $A=\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & k \\ -1 & k & 4 \end{bmatrix}$ where k is real and $k \in (a,b)$. Find the values of a and b, such that matrix A has a unique Cholesky decomposition of the form $A=LL^T$, where L is a lower triangular matrix.	**[4]**	**CO2**
3.	Consider the IVP: $y'=y^2+x, y(0)=1$ Use Taylor's series method to compute $y(0.1)$ correct to 2 decimal places.	**[4]**	**CO3**
4.	Give the geometrical interpretation of Modified Euler's method to solve the IVP: $y'=f(x,y), y(x_0)=y_0.$	**[4]**	**CO3**
5.	Derive the explicit Bender-Schmidt recurrence formula for one dimensional heat equation.	**[4]**	**CO4**

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6. Find the missing terms in the following table: **[10]** **CO1**

x:	$a-2$	$a-1$	a	$a+1$	$a+2$
$f(x)$:	3	?	-1	?	3

7. Compute the definite integral **[10]** **CO1**

$$I=\int_{-2}^{2} max\{|x^3|, x^2\} dx$$

using Simpson's rule by dividing the interval $[-2,2]$ into 4 equal parts. Also

	compare the result with the actual value of the integral and calculate the absolute error in the calculated value of I.		
8.	An iterative method to find the root of the equation $f(x)=0$ is such that it fails if $f'(x_i)\approx 0$ for some x_i in the interval bracketing the root. Identify the method and use it to find the cube root of 10 correct to three decimal places.	[10]	CO2
9.	Consider the initial value problem: $\frac{dy}{dx}=f(x,y), y(x_0)=y_0$ Suppose $f(x,y)=g(x)$ and $y(x_1=x_0+h)$ is calculated using Runge-Kutta method of fourth order. Show that this method eventually reduces to Simpson's rule of numerical integration for $f(x,y)$ with step-size $\frac{h}{2}$. **OR** The fourth order Runge−¿Kutta method $u_{j+1}=u_j+\frac{1}{6}[K_1+2K_2+2K_3+K_4]$ is used to solve the initial value problem: $\frac{du}{dt}=u, u(0)=\alpha$. If $u(1)=1$ is obtained by taking the step size $h=1$, then find the value of α.	[10]	CO3
SECTION C **(Q10 is compulsory and Q11 has internal choice)**			
10.	**a**.Solve $u_t=5u_{xx}$ with $u(0,t)=0; u(5,t)=60$ and $u(x,0)=\begin{cases}20x & for\ 0<x\le 3\\ 60 & for\ 3<x\le 5\end{cases}$; for five time steps taking $h=1$ by using Bender-Schmidt method. **b**. Use finite-difference method to determine the value of $y(0.5)$ for the boundary value problem $y''+y+1=0, 0\le x\le 1$ with the conditions $y(0)=y(1)=0$. Take $h=\frac{1}{4}$.	[10+10]	CO3 CO4
11.	**a**. Consider the IVP: $\frac{dy}{dx}=y-x, y(0)=1$. Using Taylor's series method find an approximate solution $y\approx f(x)$ of the given IVP. Assuming $y=f(x)+\epsilon(x)$ as the accurate solution of the given IVP, try to find the solution y accurately. **b**. Using Galerkin's method, compute the value of $y(0.5)$ given the boundary value problem defined by $y''+y+x=0, 0<x<1$ with the conditions $y(0)=y(1)=0$. **OR**	[10+10]	CO3 CO4

11.	**a**. Solve $u_t = u_{xx}$ with $u(x,0)=0; u(0,t)=0$ and $u(1,t)=t$. Compute u for $t=1/8$ in two time steps, using Crank-Nicolson's method. **b**. Using Galerkin's method, compute the value of $y(0.5)$ given the boundary value problem defined by $y'' + y - x^2 = 0, 0<x<1$ with the conditions $y(0)=y(1)=0$.	**[10+10]**	**CO3** **CO4**

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Instructions:
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	SECTION A **(Attempt all questions)**		
1.	Let $f(x)=x^3+1$. Find the factorial notation of $f(x)$.	**[4]**	**CO1**
2.	Derive the implicit Crank-Nicolson recurrence formula for one dimensional heat equation.	**[4]**	**CO4**
3.	Derive Modified Euler's method to solve the IVP: $y' = f(x,y), y(x_0)=y_0$.	**[4]**	**CO3**
4.	Use Taylor's series approximation method to compute $y(0.1)$ correct to 2 decimal places if given that : $y' = y^2 - x, y(0)=1$	**[4]**	**CO3**
5.	For what values of a and b the symmetric matrix $A=\begin{bmatrix}1 & 2 & k\\ 2 & 6 & 1\\ k & 1 & 2\end{bmatrix}$ where k is real and $k \in (a,b)$ has a unique Cholesky decomposition of the form $A=LL^T$, where L is a lower triangular matrix.	**[4]**	**CO2**
	SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**		
6.	Compute the following definite integral using Simpson's rule by dividing the interval $[-2,2]$ into 4 equal parts.	**[10]**	**CO1**

$$I=\int_{-2}^{2} max\{|x^3|, x^4\}\,dx$$

Also compare the result with the actual value of the integral and calculate the absolute error in the calculated value of I.

7. Find the missing terms in the following table: **[10]** **CO1**

x:	$a-2$	$a-1$	a	$a+1$	$a+2$
$f(x)$:	3	0	?	0	?

8. An iterative scheme to find the zero of $f(x)$ is such that it converges quickly if $1/f'(x_i)\approx 0$ for some x_i in the interval bracketing the root. Identify the method and use it to find $(11)^{\frac{1}{3}}$ correct to 3 decimal places. **[10]** **CO2**

9. Suppose the Runge−¿Kutta method (of orderIV) **[10]** **CO3**

$$u_{j+1}=u_j+\frac{1}{6}[K_1+2K_2+2K_3+K_4]$$

is applied to solve the initial value problem: $\frac{du}{dt}=2u, u(0)=\alpha$.

If $u(1)=1/2$ is obtained by taking the step size $h=1$, then find the value of α.

OR

Consider the IVP: $\frac{dy}{dx}=f(x,y), y(x_0)=y_0$

Suppose $f(x,y)=g(x)$ and $y(x_1=x_0+h)$ is calculated using Runge-Kutta method of fourth order. Show that this method eventually reduces to Simpson's rule of numerical integration for $f(x,y)$ with step-size $\frac{h}{2}$.

SECTION C

(Q10 is compulsory and Q11 has internal choice)

10. **[10+10]**

a. Solve $u_t=5u_{xx}$ with $u(0,t)=0; u(5,t)=120$ and $u(x,0)=\begin{cases}40x & for\ 0<x\le 3\\ 120 & for\ 3<x\le 5\end{cases}$; for five time steps taking $h=1$ by using Bender-Schmidt method. **CO3**

b. Consider the BVP defined by $y''+y-x=0, 0<x<1$ with the conditions $y(0)=y(1)=0$. Using Galerkin's method to compute the value of $y(0.5)$. **CO4**

11. **[10+10]**

a. Using Taylor's series method find an approximate solution $y\approx f(x)$ of the IVP: $\frac{dy}{dx}=x-y, y(1)=0$. Assuming $y=f(x)+\epsilon(x)$ as the accurate solution of the given IVP, try to find the solution y accurately. **CO3**

b. Compute the value of $y(0.5)$ using finite-difference method for the BVP **CO4**

	$y'' + y - 1 = 0, 0 \le x \le 1$ with the conditions $y(0) = y(1) = 0$. Take $h = \frac{1}{4}$. **OR**		
11.	**a**. Solve $u_t = u_{xx}$ with $u(x,0) = 0; u(0,t) = 0$ and $u(1,t) = 2t$. Compute u for $t = 1/8$ in two time steps, using Crank-Nicolson's method. **b**. Compute the value of $y(0.5)$ using Galerkin's method for the given BVP defined by $y'' + y + x^2 = 0, 0 < x < 1$ with the conditions $y(0) = y(1) = 0$.	**[10+10]**	**CO3** **CO4**