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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme: B.Tech. (APE-GAS, ME,MECHATRONICS,ADE, Chemical)

Course Name: Mathematics-III

Course Code: MATH-2003

No. of page/s: 02

Semester – I

Max. Marks : 100

Duration : 3 Hrs

Instructions:

Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt all questions from **Section C** (each carrying 20 marks).

SECTION A
(Attempt all questions)

1.	Verify that $X = \begin{pmatrix} 2e^{5t} \\ e^{5t} \end{pmatrix}$ is the solution of the system $\frac{dX}{dt} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} X$.	[4]	CO1
2.	Use Cauchy integral formula to evaluate $\oint_C \frac{e^{1/z}}{z} dz$, where the contour C is the circle $ z = 1$ traced counterclockwise.	[4]	CO2
3.	Use Cauchy Residue theorem to evaluate $\oint_C \frac{P(z)}{(z-1)^{15}} dz$, where $P(z) = \sum_{n=1}^{15} z^n$ and the contour C is $z = e^{i2\theta}, 0 \leq \theta \leq 2\pi$.	[4]	CO3
4.	Let C be a closed differentiable contour oriented counterclockwise and let $\oint_C \bar{z} dz = A$ where $z = x + iy$. Evaluate $\oint_C (x + y) dz$ in terms of A .	[4]	CO3
5.	Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x+2y}$.	[4]	CO4

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Find the general and singular solution of Clairaut's equation $y = px - e^p$.	[10]	CO1
7.	Suppose $\phi_k(t) = \begin{bmatrix} \phi_{1k}(t) \\ \phi_{2k}(t) \\ \vdots \\ \phi_{nk}(t) \end{bmatrix}$, $k = 1, 2, 3, \dots, n$ are the solutions of $\frac{dX}{dt} = AX$. If the Wronskian $W(t_0) = 0$ at some $t_0 \in [a, b]$ then prove that $\phi_k, k = 1, 2, 3, \dots, n$ are linearly dependent on $a \leq t \leq b$.	[10]	CO1

8.	If $u(x, y) = \frac{1}{2} \log_e(x^2 + y^2)$, find $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic by using Milne Thomson method.	[10]	CO2
9.	If a is any complex number with $ a < 1$ and C is the simple closed curve $ z = 1$ oriented counterclockwise, then find the value of $\frac{(1- a ^2)}{\pi} \int_C \frac{ dz }{ z+a ^2}$ OR If $P(z)$ is a polynomial and C denotes the circle $ z-a =R$. What is the value of $\oint_C P(z) d\bar{z}$?	[10]	CO3
SECTION C (Q10 is compulsory and Q11 has internal choice)			
10.	a. Construct a linear fractional transformation that maps the points $0, -1$ and ∞ onto the points $-1, -2 - i$ and i respectively. b. Find the solution of the form $u = f_1(x) f_2(y)$ satisfying $\frac{\partial u}{\partial x} - 2 \frac{\partial u}{\partial y} - u = 0$ with $u = 6e^{-3x}$ when $y = 0$.	[10+10]	CO3 CO4
11.	a. Consider the line L defined by the equation $z = (1 - i)x$ (where x is real) which divides the z -plane into two symmetrical halves S_1 and S_2 . If S_1 contains the positive quadrant then find the image of S_1 under the bilinear transformation $w(z) = \frac{z-1}{z+i}$. b. A string is stretched between two fixed points at a distance l apart. Motion is started by displacing the string in the form $y = y_0 \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the displacement at any point at a distance x from one end at time t . OR	[10+10]	CO3 CO4
11.	a. Consider the functions $f(z) = \frac{z^2 + \alpha z}{(z+1)^2}$ and $g(z) = \sinh\left(z - \frac{\pi}{2\alpha}\right)$, $\alpha \neq 0$. If the residue of $f(z)$ at its pole is equal to 1, then find a point where the function $g(z)$ is not conformal. b. If the mid point of a string of length l with fixed end points $x = 0$ and $x = l$ is taken to a small height h and released from the rest at time $t = 0$. Then find the displacement function $y(x, t)$.	[10+10]	CO3 CO4

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Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks); attempt all questions from **Section C** (each carrying 20 marks).

SECTION A

(Attempt all questions)

1.	Show that $X = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$ will satisfy the system $\frac{dX}{dt} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} X$.	[4]	CO1
2.	Evaluate $\oint_C \frac{\cos\left(\frac{1}{z}\right)}{z} dz$, where the contour C is the circle $ z = 1$ traced counterclockwise, using Cauchy's integral formula	[4]	CO2
3.	Evaluate $\oint_C (x - y) dz$ in terms of A , where C be a closed differentiable contour oriented counterclockwise and let $\oint_C \bar{z} dz = A$ where $z = x + iy$.	[4]	CO3
4.	Find the particular integral of $(D^2 - 2DD' + D'^2)z = e^{x-2y}$.	[4]	CO4
5.	Evaluate $\oint_C \frac{P(z)}{(z-1)^{15}} dz$, where $P(z) = \sum_{n=1}^{15} z^n$ and the contour C is $z = e^{i\theta}, 0 \leq \theta \leq 4\pi$ using Cauchy's Residue theorem.	[4]	CO3

SECTION B

(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Find the general and singular solution of the equation $y = 4px^2 - 2e^{2p}$.	[10]	CO1
7.	If $v(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$, find $v(x, y)$ such that $f(z) = u(x, y) + iv(x, y)$ is analytic by using Milne Thomson method.	[10]	CO2

8.	<p>Consider the system $\frac{dX}{dt} = AX$. If $\phi_k(t) = \begin{bmatrix} \phi_{1k}(t) \\ \phi_{2k}(t) \\ \vdots \\ \phi_{nk}(t) \end{bmatrix}$, $k = 1, 2, 3, \dots, n$ are the solutions of this system. Then prove that if the Wronskian $W(t_0)$ vanishes at some $t_0 \in [a, b]$ then $\phi_k, k = 1, 2, 3, \dots, n$ are linearly dependent on $t \in [a, b]$.</p>	[10]	CO1
9.	<p>If C is the simple closed curve $z = 1$ oriented counterclockwise, b is any complex number with $b < 1$ then find the value of</p> $\frac{(1- b ^2)}{\pi} \int_C \frac{ dz }{ z-b ^2}$ <p style="text-align: center;">OR</p> <p>Evaluate $\oint_C P(z) d\bar{z}$ where, $P(z)$ is a polynomial and C denotes the circle $z+b =R$.</p>	[10]	CO3
<p>SECTION C (Q10 is compulsory and Q11 has internal choice)</p>			
10.	<p>a. Find the functions $f_1(x)$ and $f_2(x)$ such that $u = f_1(x) f_2(y)$ is the solution of $\frac{\partial u}{\partial x} - 4 \frac{\partial u}{\partial y} - u = 0$ with $u(x, 0) = 6e^{-3x}$.</p> <p>b. Construct a bilinear transformation that maps the points 0, 1 and ∞ onto the points $-1, 2+i$ and i respectively.</p>	[10+10]	CO3 CO4
11.	<p>a. Consider the line L defined by the equation $z = (1-i)x$ (where x is real) which divides the z-plane into two symmetrical halves S_1 and S_2. If S_1 contains the negative quadrant then find the image of S_1 under the bilinear transformation $w(z) = \frac{z-1}{z+i}$.</p> <p>b. A string is stretched between two fixed points at a distance $2l$ apart. Motion is started by displacing the string in the form $y = y_0 \sin \frac{\pi x}{2l}$ from which it is released at time $t = 0$. Find the displacement at any point at a distance x from one end at time t.</p> <p style="text-align: center;">OR</p>	[10+10]	CO3 CO4

<p>11.</p>	<p>a. Consider the functions $f(z) = \frac{z^3 + \alpha z}{(z+1)^2}$ and $g(z) = \sinh\left(z - \frac{\pi}{2\alpha}\right)$, $\alpha \neq 0$. If the residue of $f(z)$ at its pole is equal to 6, then find a point where the function $g(z)$ is not conformal.</p> <p>b. If the mid point of a string of length $2l$ with fixed end points $x = 0$ and $x = 2l$ is taken to a small height h and released from the rest at time $t = 0$. Then find the displacement function $y(x, t)$.</p>	<p>[10+10]</p>	<p>CO3</p> <p>CO4</p>
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