| Name: <br> Enrolment No: |  |  |  |
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| \left. UNIVERSITY OF PETROLEUM AND ENERGY STUDIES   <br> End Semester Examination, December 2018   $\right]$ |  |  |  |
| ALL THREE QUESTIONS ARE COMPULSORY (Total 100 Marks) |  |  |  |
| Q1 | Consider the following ODE-BVP: $\frac{d^{2} y}{d x^{2}}+2 y x=x y \int_{x=0}^{1}\left(y^{2} e^{x}\right) d x ; 0 \leq \mathrm{x} \leq 1$ <br> with $y(x=0)=4 ; \quad x=0$ <br> and $y(x=1)=9$ <br> Use $N=2$ (therefore, $N+1=3$ FD points, $x_{1}, x_{2}$ and $x_{3}$ ) <br> (a) Write the values of $y_{1}$ and $y_{3}$ <br> (b) Use Simpson's rule (with $h=0.5$ ) to evaluate the integral on the right hand side. Compute all the values and give a simple final expression <br> (c) Use the finite difference technique to obtain an equation for $y_{2}$. Simplify as much as possible (10) | $\begin{gathered} 30 \\ \text { marks } \end{gathered}$ | CO5 |


|  | (d) Check if $y_{2}=0.02345$ satisfies this equation (05) |  |  |
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| Q2 | Consider the following ODE-IVP involving two variables, $y_{1}(\mathrm{t})$ and $y_{2}(\mathrm{t})$ : <br> i <br> with $\boldsymbol{y}(t=0)=\boldsymbol{y}_{0}=\left[\begin{array}{ll}3 & 0\end{array}\right]^{\mathrm{T}}$ <br> (a) Apply the implicit Euler technique (for one step only, i.e., from $t=0$ to $t=h$ $=0.1$ ) to obtain your answers in the following form: $\begin{equation*} F(y) \equiv i \tag{10} \end{equation*}$ <br> (b) Now use the Newton-Raphson-Kantarovich technique WITHOUT using inverses in any part of this question to obtain <br> i <br> Do one NRK iteration only. Obtain numerical answers (10) <br> (c) Plug in your answers in part (b) of this question (i.e., into the $\boldsymbol{F}(\boldsymbol{y})=\mathbf{0}$ equation) and see what you get (05) <br> (d) Comment on your answer to part (c) of this question. (10) | $\begin{gathered} 35 \\ \text { marks } \end{gathered}$ | $\begin{aligned} & \mathrm{CO} 2, \\ & \mathrm{CO} 3 \\ & \mathrm{CO} 4 \end{aligned}$ |
| Q3 | Consider the third order $(q-1=3)$ implicit Hermite algorithm (in Table 5.1) of integrating ODE-IVPs for a single variable, $y(x)$, with $\begin{aligned} & \alpha_{0}=\frac{1}{2} \\ & \alpha_{1}=\alpha_{3}=\alpha_{4}=\ldots=0 \\ & \alpha_{2}=\frac{1}{2} \\ & \beta_{0}=\frac{-1}{4} \end{aligned}$ | $\begin{gathered} 35 \\ \text { marks } \end{gathered}$ | $\begin{aligned} & \mathrm{CO} 2, \\ & \mathrm{CO} 3 \end{aligned}$ |

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\begin{aligned}
& \beta_{1}=\beta_{3}=\beta_{4}=\ldots=0 \\
& \beta_{2}=\frac{1}{4}
\end{aligned}
$$

(a) Write down the algorithm for $y_{n+1}$ in terms of $y_{i}$ and $y_{i}^{\prime}$
(b) Using $\frac{d y}{d t}=\lambda y, y(t=0)=y_{0}$, obtain the characteristic equation for $\mu$ and solve for $\mu_{\mathrm{i}}$ [Hint: Note that you will get two values of $\mu_{\mathrm{i}}$ ]
(c) Which of these two roots is the genuine root and which is the spurious root
(d) What is the requirement of stability for this problem?

