Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2018

Programme Name: B. Tech CivilE-Sz-Infra — **Semester : III**
Course Name : Applied Numerical Methods — **Time : 03 hrs**
Course Code : MATH 3002 — **Max. Marks : 100**
Nos. of page(s) : 03

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Instructions: Attempt all questions from **Section A** (each carrying 4 marks); **Section B** (each carrying 10 marks); **Section C** (each carrying 20 marks).

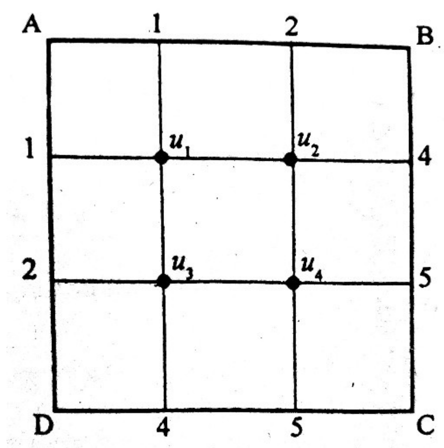
SECTION A
(Attempt all questions)

Q. No.		Marks	CO
1.	Estimate the missing figures in the following table: <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>2</td><td>−¿</td><td>7</td><td>−¿</td><td>32</td></tr></table>	**[4]**	**CO1**
2.	Establish the operator relation $E \equiv e^{hD}$, where E and D denote the Shifting and Differential operators respectively. (h is the step-length).	**[4]**	**CO1**
3.	Let $x_0 = 1.5$ be the initial approximation of a root of the equation $x^2 + \log_e x - 2 = 0$. Find the first approximate root of the equation using fixed-point iteration method (iteration method).	**[4]**	**CO2**
4.	Decompose the matrix $A = \begin{bmatrix} 4 & 3 \\ 6 & 3 \end{bmatrix}$ in "**LU**" form by Crout's decomposition method.	**[4]**	**CO2**
5.	Performing two iteration by Picard method, find an approximate value of $y(0.2)$ for the differential equation $\frac{dy}{dx} = x - y$,with the initial condition $y(0) = 1$.	**[4]**	**CO3**

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Let a function $f(x)$ be known for $(n+1)$ arguments, namely $x_0, x_1, x_2, \ldots, x_n$,which are not necessarily equi-spaced, and $y_j = f(x_j)$, for $j = 0, 1, 2, \ldots, n$ be the corresponding entries. Derive Newton's divided difference interpolation formula to construct an approximate polynomial $\phi(x)$ of degree less than equal to n which satisfies the conditions $\phi(x_j) = f(x_j) = y_j$, for $j = 0, 1, 2, \ldots, n$.	**10**	**CO1**

7.	Let x_0 be an initial approximation of the root of the equation $f(x)=0$, where $f(x)$ is differentiable at $x=x_0$. Derive the first approximate root x_1 by Newton-Raphson method. Hence find the first approximate root of the equation $$\log_e x + x - 3 = 0,$$ considering $x_0=2$.	**10**	**CO2**
8.	Use fourth order Runge-Kutta method to solve for $y(1.2)$, considering step-length $h=0.1$ for the differential equation $$\frac{dy}{dx}=x^2+y^2,$$ with the initial condition $y(1)=1.5$.	**10**	**CO3**
9.	Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Gauss-Seidel method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)}=1, x_2^{(0)}=0, x_3^{(0)}=1$. Perform three iterations. $x_1+5x_2+3x_3=28$ $12x_1+3x_2-5x_3=1$ $3x_1+7x_2+13x_3=76.$ **OR** Using Gauss-Jacobi method, solve the following system of equations starting with initial solution as $x_1^{(0)}=\frac{9}{5}, x_2^{(0)}=\frac{4}{5}, x_3^{(0)}=\frac{6}{5}$. Perform three iterations. $5x_1-x_2=9$ $-x_1+5x_2-x_3=4$ $-x_2+5x_3=-6.$	**10**	**CO2**
SECTION-C (Q10 is compulsory; Q11 has two parts, Q11.A and Q11.B, both have internal choices)			
10.A	Evaluate the integration $\int_0^{0.8} \frac{\sin x}{x}\,dx$, using (i) Simpson's $\frac{1}{3}$ rule and (ii) Simpson's $\frac{3}{8}$ rule, by dividing the intervals $[0, 0.8]$ into six equal parts.	**10**	**CO1**
10.B	Consider a matrix $A=\begin{bmatrix} 4 & 2 & 6 \\ 2 & 82 & 39 \\ 6 & 39 & 26 \end{bmatrix}$. i. Is it possible to find LU-decomposition for the matrix A in form of $A=LL^T$, where L is a lower triangular matrix, by Cholesky method? ii. If yes, decompose the matrix in form of $A=LL^T$. iii. Hence, find the solution of the following system by Cholesky method. $4x_1+2x_2+6x_3=16$	**10**	**CO2**

	$2x_1+82x_2+39x_3=206$ $6x_1+39x_2+26x_3=113$		
11.A	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $$\frac{dy}{dx}=-ky, \text{ where } k=0.01.$$ Given that $x_0=0$ and $y_0=100$. Determine how much substance will remain at the moment $x=100$, using Modified Euler's method with the step-length $h=50$. **OR** Solve the elliptic equation $u_{xx}+u_{yy}=0$ for the following square mesh with boundary values as shown in the figure by Liebmann's iteration process. Perform five iterations.  	**10**	**CO3**
11.B	Solve $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x)$ taking $h=1$ and employing Bender-Schmidt method. Continue the solution through five time steps. **OR** Using Crank-Nicholson's method, solve $u_{xx}=16u_t, 0<x<1, t>0$, given that $u(x,0)=0, u(0,t)=0, u(1,t)=50t$. Compute u for two steps in t direction taking $h=\frac{1}{4}$.	**10**	**CO3**

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SECTION A
(Attempt all questions)

Q. No.		**Marks**	**CO**
1.	Estimate the missing figures in the following table: x: 1, 2, 3, 4, 5 y: 2, 5, 7, −¿, 32	**[4]**	**CO1**
2.	Establish the operator relation $\Delta - \nabla \equiv \Delta \nabla$, where Δ and ∇ denote the Newton forward and Backward operators respectively.	**[4]**	**CO1**
3.	Find the value of $(17)^{\frac{1}{3}}$ after four iteration, considering the initial interval as $[2,3]$ by Bisection method.	**[4]**	**CO2**
4.	Decompose the matrix $A=\begin{bmatrix} 2 & 3 \\ 6 & 3 \end{bmatrix}$ in "**LU**" form by Crout's decomposition method.	**[4]**	**CO2**
5.	Classify the partial differential equation $a^2 u_{xx} = u_{tt}$, where a is constant.	**[4]**	**CO3**

SECTION B
(Q6-Q8 are compulsory and Q9 has internal choice)

6.	Let a function $f(x)$ be known for $(n+1)$ arguments, namely $x_0, x_1, x_2, \ldots, x_n$, which are not necessarily equi-spaced, and $y_j = f(x_j)$, for $j=0,1,2,\ldots,n$ be the corresponding entries. Derive Lagrange interpolation formula to construct an approximate polynomial $L(x)$ of degree less than equal to n which satisfies the conditions $L(x_j) = f(x_j) = y_j$, for $j=0,1,2,\ldots,n$.	**10**	**CO1**
7.	Let x_0 be an initial approximation of the root of the equation $f(x)=0$, where $f(x)$ is	**10**	**CO2**

	differentiable at $x=x_0$. Derive the first approximate root x_1 by Newton-Raphson method. Hence find the first approximate root of the equation $$2x+\log_{10}x-7=0,$$ considering $x_0=3.5$.		
8.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $$\frac{dy}{dx}=-ky, \text{ where } k=0.01.$$ Given that $x_0=0$ and $y_0=100$. Determine how much substance will remain at the moment $x=100$, using Modified Euler's method with the step-length $h=50$.	**10**	**CO3**
9.	Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Gauss-Jacobi method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)}=1, x_2^{(0)}=0, x_3^{(0)}=1$. Perform three iterations. $x_1+5x_2+3x_3=28$ $12x_1+3x_2-5x_3=1$ $3x_1+7x_2+13x_3=76.$ **OR** Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then using Gauss-Seidal method, solve the following system of equations starting with initial solution as $x_1^{(0)}=\frac{9}{5}, x_2^{(0)}=\frac{4}{5}, x_3^{(0)}=\frac{6}{5}$. Perform three iterations. $-x_1+5x_2-x_3=4$ $5x_1-x_2=9$ $-x_2+5x_3=-6.$	**10**	**CO2**
SECTION-C **(Q10 is compulsory; Q11 has two parts, Q11.A and Q11.B, both have internal choices)**			
10.A	Evaluate the integration $\int_0^{\frac{\pi}{2}} \sqrt{1-k\sin^2\phi}\,d\phi, k=0.162$, by Simpson's $\frac{1}{3}$ rule, dividing the interval $0\le\phi\le\frac{\pi}{2}$ into six equal subintervals.	**10**	**CO1**
10.B	Consider a matrix $A=\begin{bmatrix}4 & 2 & 6\\ 2 & 82 & 39\\ 6 & 39 & 26\end{bmatrix}$. i. Is it possible to find LU-decomposition for the matrix A in form of $A=LL^T$, where L is a lower triangular matrix, by Cholesky method? ii. If yes, decompose the matrix in form of $A=LL^T$. iii. Hence, find the solution of the following system by Cholesky method. $4x_1+2x_2+6x_3=16$ $2x_1+82x_2+39x_3=206$	**10**	**CO2**

	$6x_1+39x_2+26x_3=113$		
11.A	Use fourth order Runge-Kutta method to solve for $y(1.2)$, considering step-length $h=0.1$ for the differential equation $$\frac{dy}{dx}=x^2+y^2,$$ with the initial condition $y(1)=1.5$. **OR** Solve the Poisson's equation $u_{xx}+u_{yy}=-10(x^2+y^2+10)$ over the square mesh with sides $x=0, y=0, x=3, y=3$ with $u=0$ on the boundary and mesh length 1. Perform three iterations by Gauss Seidal method to solve the linear equations in u. .	**10**	**CO3**
11.B	Solve $\frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t)=0, u(4,t)=0, u(x,0)=x(4-x)$ taking $h=1$ and employing Bender-Schmidt method. Continue the solution through five time steps. **OR** Using Crank-Nicholson's method, solve $u_{xx}=16u_t, 0<x<1, t>0$, given that $u(x,0)=0, u(0,t)=0, u(1,t)=50t$. Compute u for two steps in t direction taking $h=\frac{1}{4}$.	**10**	**CO3**

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