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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, April, 2017

Program/course: B. Tech ASE | **Semester – VIII**
Subject: Finite Element Analysis | **Max. Marks : 100**
Code : ASEG483 | **Duration : 3 Hrs**
No. of page/s: 02

Section-A

1) Distinguish between essential and natural boundary conditions. [04]

2) What are the conditions for a problem to be axisymmetric? Write down the stress-strain relationship matrix for an axisymmetric triangular element. [04]

3) What are the four basic elastic equations? [04]

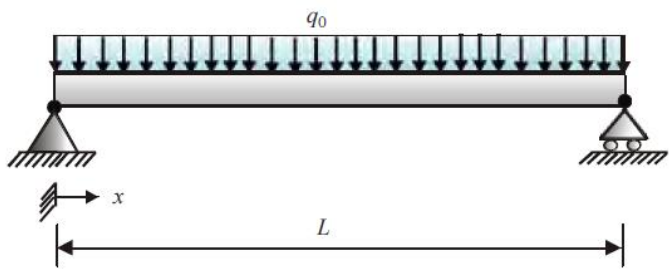
4) What is meant by discretization and assemblage? During discretization, mention the places where it is necessary to place a node? [04]

5) What is a truss? Define with a specific truss example how to calculate the total potential energy of the system. [04]

Section-B

6) Consider a simply supported beam under uniformly distributed load as shown in figure below. The governing differential equation and the boundary conditions are given by,

$$EI\frac{d^4v}{dx^4} - q_0 = 0; \qquad v(0) = 0, \frac{d^2v}{dx^2}(0) = 0, v(L) = 0, \frac{d^2v}{dx^2}(L) = 0$$



Find the approximate solution using <u>any one</u> of the following methods

a) The point collocation technique at $x = L/2$.
b) Least square method

Assume a one parameter trial solution: $v(x) \approx \hat{v}(x) = c_1 sin(\pi x/L)$ [10]

7) Define the following;
   (i) Stiffness matrix
   (ii) Functional
   (iii) Shape function [10]

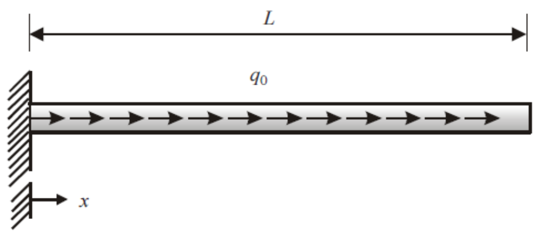
8) Solve the following equation using a two-parameter trial solution by the Rayleigh-Ritz method,

$$\frac{dy}{dx} + y = 0, \qquad y(0) = 1 \qquad [10]$$

9) What do you mean by weak form of the differential equation? State the advantages of the weak form over the weighted residual method. [10]

Section-C

10) Consider a uniform rod subjected to a uniform axial load as shown in figure. The deformation of the bar is governed by the differential equation,

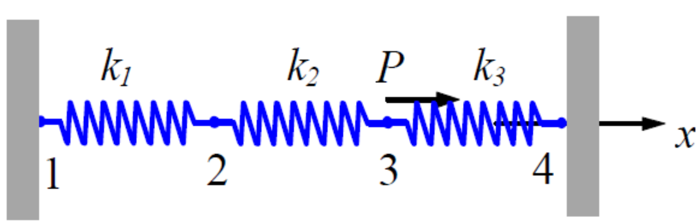


$$AE\frac{d^2u}{dx^2} + q_0 = 0$$

with the boundary conditions, $u(0) = 0, \quad \frac{du}{dx}_{x=L} = 0$

Find an appropriate solution to this problem using the Weighted Residual Method. [20]

11) For the spring system shown below,
k_1 = 200 N / mm, k_2 = 100 N / mm, k_3 = 200 N / mm
P = 10 N (applied at point 3). The fixed boundary leads to the displacement $U_1 = U_4 = 0$



Find: (a) Global stiffness matrix
(b) Displacements of nodes 2 and 3
(c) Reaction forces at nodes 1 and 4
(d) Force in the spring 2 [20]

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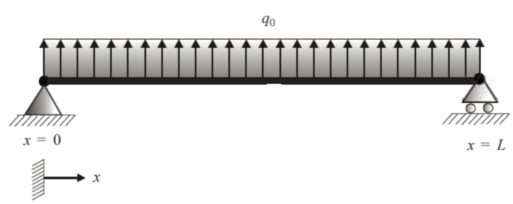
Section-A

1) What is the difference between static and dynamic analysis? Why polynomials are generally used as shape function? [4]

2) Define initial and boundary value problems. [4]

3) Consider a simply supported beam under uniformly distributed load q_0 as shown in figure. For the deformation $v(x)$, we have



The strain energy $U = \int_0^L \frac{1}{2} EI \left(\frac{d^2y}{dx^2}\right)^2 dx$

The potential of the external forces is $V = -\int_0^L q_0 v \, dx$

Find the appropriate approximation for deformation using the Principle of Stationary Total Potential (PSTP). [4]

4) State some of the advantages FEA has compared to other approximation methods. [4]

5) What do you mean by Weighted Residual Method? Define, under this method, the steps involved to find the approximate solution to a differential equation. [4]

Section-B

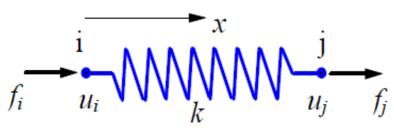
6) Deformation of a uniform rod of length L with uniform axial load q is given by,

$$AE\frac{d^2y}{dx^2} + q = 0$$

with boundary conditions: y(0) = 0, $AE\left.\frac{dy}{dx}\right|_L = 0$

Derive the weak form taking into consideration all the boundary conditions. [10]

7) Consider a single spring element with the given notations,



Two nodes:	i	j
Nodal displacements:	u_i	u_j
Nodal forces:	f_i	f_j
Spring constant (stiffness)	k	

Using the spring-displacement relationship, derive the expression, **ku** = **f**
where, k = (element) stiffness matrix, u = (element nodal) displacement vector
f = (element nodal) force vector [10]

8) Define types of elements with proper schematic for different dimensions of space. Give examples for each type of elements. [10]

9) Describe briefly the Method of Weighted Residuals (MWR). Furthermore, explain the application of MWR in the following methods,
(i) Method of Point Collocation
(ii) Method of Collocation by Sub-Regions [10]

Section-C

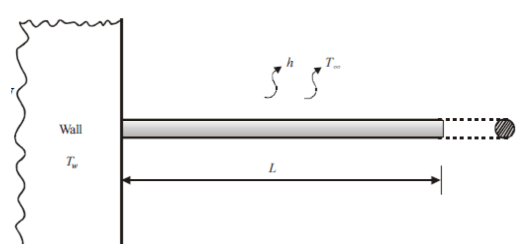
10) Derive the Euler-Lagrange equation for a functional given by,

$$I(u) = \int_a^b F\left(u, \frac{du}{dx}, x\right) dx$$

Thus, obtain the corresponding Euler-Lagrange for the functional given below,

$$I = \frac{1}{2}\int_0^L \left[\propto \left(\frac{dy}{dx}\right)^2 - \beta y^2 + ryx^2\right] dx - y(L)$$ [20]

11) Consider a 1 *mm* diameter, 50 *mm* long aluminum pin fin as shown in the figure below that is used to enhance the heat transfer from a surface wall maintained at 300°C. The governing differential equation and the boundary conditions are given by,



$$k\frac{d^2T}{dx^2} = \frac{Ph}{A_c}(T - T_\infty); \qquad T(0) = T_w = 300^oC, \qquad \left.\frac{dT}{dx}\right|_{(L)} = 0$$

Let k = 200W/m/°C for aluminum, h = 20 W/m^2°C, T_∞ = 30°C. Estimate the temperature distribution in the fin at 10 equal points using the Galerkin residual method using an appropriate polynomial trial function. [20]