UNIVERSITY OF PETROLEUM & ENERGY STUDIES

DEHRADUN

End Semester Examination – May, 2018

Program/course: M.Tech. (Computational Fluid Dynamics) **Semester – II**
Subject: Advanced Computing Techniques **Max. Marks : 100**
Code: ASEG7016 **Duration : 3 Hrs**
No. of page/s: 4

Section –A

Note: All questions are compulsory.

Q1: Explain Newton's method for estimating the roots of an equation. How is it different from bisection method? [**5**]

Q2: Using Lagrange interpolating polynomials, approximate the following function:

$$f(x) = e^{2x} \sin 3x$$

using the nodes $x_0 = 0, x_1 = 0.3, x_2 = 0.6, n = 2$ [**5**]

Q3: Discuss the importance of vector norms in numerical computations. For the following linear system of equations

$$3.3330\, x_1 + 15920 x_2 - 10.333\, x_3 = 15913$$
$$2.2220\, x_1 + 16.710\, x_2 - 9.6120\, x_3 = 28.544$$
$$1.5611\, x_1 + 5.1791\, x_2 + 1.6852\, x_3 = 8.4254$$

which has approximate solution $\tilde{\mathbf{x}} = (\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3})^{\boldsymbol{t}} = (1.2001, 0.99991, 0.92538)^t$ and the exact solution as $\mathbf{x} = (1,1,1)^t$, determine l_2 and l_∞ norms. [**5**]

Q4: Cite out differences between Gauss-Seidel and Successive Over-Relaxation schemes for solving a system of linear equations. On what basis the value of ω is decided? [**5**]

Section – B

Questions 5-8 are compulsory. Question 9 has an internal choice

Q5: What do you mean by spectral radius of a matrix A. Find the spectral radius of the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

[5]

Q6: Outline various differences between a subroutine and a function in FORTRAN 90? When a subroutine is called, how the data is passed from the calling program to the subroutine, and how the results of the subroutine are returned to the calling program? Explain this by writing a code to find the maximum entry in a 1D array (hint: use both the function & subroutine to write this). [**5**]

Q7: A particle of mass m moving through a fluid is subjected to a viscous resistance R, which is a function of the velocity v. The relationship between the resistance R, velocity v, and time t is given by the equation:

$$t = \int_{v(0)}^{v(t)} \frac{m}{R(u)} du$$

Suppose that $R(u) = -v\sqrt{v}$ for a particular fluid, where R is in newtons and v is in m/s. If $m = 10$ kg and $v(0) = 10$ m/s, approximate the time required for the particle to slow to $v = 5$ m/s. [**10**]

Q8. Explain Gauss elimination with scaled partial pivoting for solving a system of linear algebraic equations. Show how this scheme may be used to solve following system of linear equations (first calculate the roots and then write the code): [**10**]

$$2.12x_1 - 2.12x_2 + 51.3x_3 + 100x_4 = \pi$$
$$0.333x_1 - 0.333x_2 - 12.2x_3 + 19.7x_4 = \sqrt{2}$$
$$6.19x_1 + 8.20x_2 - 1.00x_3 - 2.01x_4 = 0$$
$$-5.73x_1 + 6.12x_2 + x_3 - x_4 = -1$$

Q9a: At a time $t = 0$, a tank contains Q_0 lb (1 lb = 0.45 Kg) of salt dissolved in 100 gal (1 gallon = 3.785 liter) of water (see the figure). Assume that water containing 1/4 lb of salt/gal is entering the tank at a rate of r gal/min, and that the well-stirred mixture is draining from the tank at the same rate. This flow process is represented by following ODE:

$$\frac{dQ}{dt} + \frac{rQ}{100} = \frac{r}{4}$$

where $Q(t)$ is the amount of salt in the tank at time t. If $r = 3$ and $Q_0 = 75$ lb, find the time T after which the salt level us 6% of Q_0. Also, find the flow rate that is required if the value of T is not to exceed 4 min. Use RK4 for the numerical solution. Choose the discretization details (e.g. time step, Δt) as needed. [**10**]

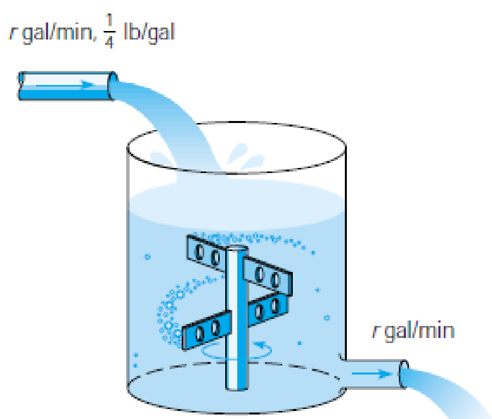

Figure: Water tank in question 9a

OR

Q9b: The secretion of hormones into the blood is often a periodic activity. If a hormone is secreted on a 24-h cycle, then the rate of change of the level of the hormone in the blood may be represented by the initial value problem:

$$\frac{dx}{dt} = \alpha - \beta\cos\frac{\pi t}{12} - kx, \qquad x(t=0) = x_0$$

where $x(t)$ is the amount of the hormone in the blood at time t, α is the average secretion rate, β is the amount of daily variation in the secretion, and k is a positive constant reflecting the rate at which the body removes the hormone from the blood. If $\alpha = \beta = 1, k = 2,$ and $x_0 = 10$, solve for $x(t)$. Use RK4 for numerical approximation of the ODE. [**10**]

Section –C

Q10 is compulsory. Question 11 has an internal choice.

Q10: The temperature $T(x,t)$ of a long, thin rod of constant cross-section and homogeneous conducting material is governed by 1D heat equation. If heat is generated in the material, for example, by resistance to current or nuclear reaction, the heat equation becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{Kr}{\rho C} = K\frac{\partial T}{\partial t}, \qquad 0 < x < l, \quad t > 0$$

where l is the length, ρ is the density, C is the specific heat, and K is the thermal diffusivity of the rod. The function $r = r(x,t,T)$ represents the heat generated per unit volume. Suppose that

$$l = 1.5\ \text{cm}, \quad K = 1.04\ \frac{\text{cal}}{\text{cm. deg. s}}, \rho = 10.6\frac{\text{g}}{\text{cm}^3}, C = 0.056\ \frac{\text{cal}}{\text{g. deg}}$$

and

$$r(x,t,T) = 5.0\ \frac{\text{cal}}{\text{cm}^3.\text{s}}$$

If the ends of the rod are kept at 0°C, then

$$T(0,t) = T(l,t) = 0, \quad t > 0$$

Suppose the initial temperature distribution is given by

$$T(x,0) = \sin\frac{\pi x}{l}, \quad 0 \le x \le l$$

Solve the above by discretizing the spatial differential operators using centered difference scheme and temporal differential operator using implicit Euler scheme. Apply the initial and boundary conditions so that the discretized equations could be written in $Ax = b$ form, where A is the stiffness matrix, x is unknown vector (here it contains the values of T) and b is the known vector. Take $h = 0.3$ and $k = 0.02$, where h and k are discretization in space and time, respectively. Write a program in FORTRAN 90 to solve the PDE with the help of Thomas Algorithm (for solving the $Ax = b$ system). [**20**]

Q11a: A 6cm × 5cm rectangular silver plate has heat being uniformly generated at each point at the rate $q = 1.5$ cal/cm^3-s. Let x represent the distance along the edge of the plate of length 6 cm and y be the distance along the edge of the plate of length 5 cm. Suppose the temperature T along the edges is kept at the following temperatures:

$$T(x,0) = x(6-x), \qquad T(x,5) = 0 \qquad 0 \le x \le 6$$
$$T(0,y) = y(5-y), \qquad T(6,y) = 0 \qquad 0 \le y \le 5$$

where the origin lies at a corner of the plate with coordinates (0,0) and the edges lie along the positive $x -$ and $y -$ axes. The steady state temperature $T = T(x,y)$ satisfies Poisson's equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -\frac{q}{K}, \qquad 0 < x < 6, \qquad 0 < y < 5$$

where K, the thermal conductivity, is 1.04 cal/cm-deg-s. Using centered finite differencing, discretize the equations and express the resulting equations in $AT = b$ form. Write a program to solve $AT = b$ using GMRES method (just call GMRES scheme with correct arguments). Use appropriate resolution to discretize the spatial differential operators. [**20**]

OR

Q11b: The air pressure $p(x,t)$ in an organ pipe is governed by the wave equation

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, \qquad 0 < x < l, 0 < t$$

where l is the length of the pipe, and c is a physical constant. If the pipe is open, the boundary conditions are given by

$$p(0,t) = p_0 \text{ and } \frac{\partial p}{\partial t}(l,t) = 0$$

Assume $c = 1, l = 1$, and the initial conditions are

$$p(x,0) = p_0 \cos 2\pi x, \text{and } \frac{\partial p}{\partial t}(x,0) = 0, \quad 0 \le x \le 1$$

Approximate the pressure for an open pipe with $p_0 = 0.9$ at $x = \frac{1}{2}$ for $t = 0.5$ and $t = 1$. Discretize both the differential operators using centered differencing and then express the system of discretized equations in $Ap = b$ form. Write a pseudocode to solve $Ap = b$ system. [**20**]

Set-II

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Section –A

Note: All questions are compulsory.

Q1. Differentiate between single step and multi-step schemes for solving initial value problems involving ODEs. How multi-step methods may be used in predictor-corrector framework? [**4**]

Q2. Define the following: [**6**]

a) "Rank" of a matrix

b) Diagonally dominant matrices

c) Symmetric and skew-symmetric matrices

Q3: Explain fixed-point iteration scheme for solving a system of non-linear equations. How can we accelerate the convergence of fixed-point iteration scheme? [**5**]

Q4: Define various types of matrix norms used in numerical linear algebra. Calculate $\|\mathbf{A}\|_\infty$ for the matrix: [**5**]

$$A = \begin{bmatrix} 4 & -1 & 7 \\ -1 & 4 & 0 \\ -7 & 0 & 4 \end{bmatrix}$$

Section – B

Questions 5-7 are compulsory. Question 8 has an internal choice

Q5. Discuss Gauss-Seidel iteration scheme for solving a system of linear equations. Solve the following system of equations

$$10x_1 - x_2 = 9$$
$$-x_1 + 10x_2 - 2x_3 = 7$$
$$-2x_2 + 10x_3 = 6$$

Using Gauss-Seidel scheme until $\frac{\|\mathbf{x}^{(k+1)}-\mathbf{x}^{(k)}\|_\infty}{\|\mathbf{x}^{(k+1)}\|_\infty} \leq 10^{-4}$, where $\mathbf{x}^{(k)}$ are iterates of the solution. Take initial guess as $\mathbf{x}^{(\mathbf{0})} = 0$. Write down a code in FORTRAN 90 to solve the above problem. [**10**]

Q6: A mathematical model that describes the 24-hr temperature profile inside a building as a function of the outside temperature, the heat generated inside the building, and the furnace heating or air conditioner cooling is given as

$$\frac{dT}{dt} = K[M(t) - T(t)] + H(t) + U(t)$$

where M is the temperature outside the building, T is the temperature inside the building, H is the additional heating rate, U is the furnace heating or air conditioner cooling rate, K is a positive constant which depends on the physical properties of the building, such as the number of doors and windows and the type of insulation, but K does not depend on $M, T,$ or t. The initial temperature is $T(t=0) = T_0 = 65$ degree Fahrenheit, $H(t) = 0.1, U(t = 1.5[70 - T(t)])$ and $M(t) = 75 - 20\cos(\pi t/12)$. The constant K is usually between 1/4 to 1/2. Using Euler method, approximate the temperature of the room with time.

[**10**]

Q7: Explain Newton's method for solving a system of non-linear equations. Write a FORTRAN 90 program to solve following system of non-linear equations:

$$3x_1 - \cos(x_2 x_3) - \frac{1}{2} = 0$$

$$x_1^2 - 625x_2^2 - \frac{1}{4} = 0$$

$$e^{-x_1 x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$$

Use $\mathbf{x}^{(0)} = (1,1-1)^t$ as the guess vector. [**10**]

Q8a: Using Gauss's elimination with partial pivoting, solve the following system of equations:

$$\pi x_1 + \sqrt{2}x_2 - x_3 + x_4 = 0$$

$$e x_1 - x_2 + x_3 + 2x_4 = 1$$

$$x_1 + x_2 - \sqrt{3}x_3 + x_4 = 2$$

$$-x_1 - x_2 + x_3 - \sqrt{5}x_4 = 3$$

Estimate the roots of the above system of equations? Also write a pseudocode to explain the steps involved in solving this problem through computer. [**10**]

OR

Q8b: Use the SOR (successive over-relaxation) method with $\omega = 1.2$ to solve the following system of linear equations with a tolerance $\text{TOL} = 10^{-3}$ in the l_∞ norm:

$$3x_1 - x_2 + x_3 = 1$$

$$3x_1 + 6x_2 + 2x_3 = 0$$

$$3x_1 + 3x_2 + 7x_3 = 4$$

with the guess vector as $\mathbf{x}^{(0)} = \mathbf{0}$. **[10]**

Section –C

Q9 is compulsory. Question 10 has an internal choice

Q9: A uniform solid rod of length l is thermally insulated along its length and its initial temperature is 0°C. One end is thermally insulated and the other end is supplied with heat at a steady rate. Estimate the subsequent temperatures at points within the rod by solving the heat transport equation:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}, \qquad 0 < x < l, \qquad t > 0$$

with initial condition $U(x, t = 0) = 0$, and the boundary conditions

$$\frac{\partial U(x = 0, t)}{\partial t} = 0 \text{ and } \frac{\partial U(x = l, t)}{\partial x} = f$$

where f is constant. Use implicit Euler to solve the above equation. Choose your discretization to take care of the accuracy and stability considerations. Write a modular code in FORTRAN 90 to solve the problem. [**20**]

Q10a: The transverse displacement y of a point at a distance x from one end of a vibrating string of length L at time t satisfies

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

with the boundary conditions as

$$y(x = 0, t) = y(x = L, t) = 0$$

and initial conditions

$$y(x, t = 0) = \frac{1}{2}x(1 - x) \text{ and } \frac{\partial y}{\partial t}(x, t = 0) = 0$$

Write a program in FORTRAN 90 to compute the displacement and velocity of the vibrating string. [**20**]

OR

Q10b: Solve the following Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)e^{xy}, \;\; 0 < x < 2, 0 < y < 1$$

with the boundary conditions

$$u(x = 0, y) = 1, u(x = 2, y) = e^{2y} \quad 0 \le y \le 1$$

$$u(x, y = 0) = 1, u(x, y = 1) = e^{x} \quad 0 \le x \le 2$$

use $\Delta x = 0.2, \Delta y = 0.1$ for discretization the space. [**20**]