Roll No: -----



UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B. Tech. (SOE)	Semester –	II
Course Name: Mathematics II	Max. Marks	: 100
Course Code: MATH 1004	Duration	: 3 Hrs
No. of page/s: 3		

Instructions:

Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 8 marks); attempt all questions from Section C (each carrying 20 marks).

	Section A (Attempt all questions)								
1.	Solve the differential equation $2\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x$.	[4]	CO1						
2.	Find the Laplace transform of $e^{-t} \cos t \cos 2t$.	[4]	CO2						
3.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & x \le 1, \\ 0 & x > 1 \end{cases}$.	[4]	CO2						
4.	A continuous random variable X has a pdf $f(x) = kx^2e^{-x}$; $x \ge 0$. Find k and mean.	[4]	CO4						
5.	Find the complete solution of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(x^2 + e^{2x} + \sin 2x)$.	[4]	CO1						
	SECTION B (Q6-Q9 are compulsory and Q10 has internal choice)								
6.	State convolution theorem and hence evaluate $L^{-1}\left(\frac{s}{\left(s^2+1\right)\left(s^2+4\right)}\right)$.	[8]	CO2						

	Solve by removal of first order derivative								
7.	Solve by removal of first order derivative $\frac{d^2 y}{dx^2} + \frac{1}{x^{1/3}} \frac{dy}{dx} + \left(\frac{1}{4x^{2/3}} - \frac{1}{6x^{4/3}} - \frac{6}{x^2}\right) y = 0.$	[8]	CO1						
8.	8. Using Laplace transform, solve the differential equation $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$.								
9.	Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience shows that 2 percent of such fuses are defective.	[8]	CO4						
	Apply Stoke's theorem to evaluate $\int_C \left[(x+y)dx + (2x-z)dy + (y+z)dz \right]$, where								
	C is the boundary of the triangle with vertices $(2,0,0)$, $(0, 3, 0)$ and $(0, 0, 6)$.								
10.	OR $\int \int \left(v \pi \hat{i} + \pi v \hat{k} \right) d\vec{k} = \pi h v r \hat{k} + h v r h v r \hat{k}$	[8]	CO3						
	Evaluate $\iint_{S} (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot d\vec{s}$, where <i>S</i> is the surface of the sphere $x^{2} + y^{2} + z^{2} = a^{2}$ in the first octant.								
	SECTION C (Q11 is compulsory and Q12 has internal choice)								
	A) Prove that $div(grad r^n) = n(n+1)r^{n-2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Hence show								
	that $\nabla^2\left(\frac{1}{r}\right) = 0$.	[10]	CO3						
11.	B) Two lines of regression are given by $5y-8x+17=0$ and $2y-5x+14=0$. If								
	$\sigma_y^2 = 16$, find i) the mean values of x and y ii) the coefficient of correlation	[10]	CO4						
	between x and y .	_							
	The demand for a particular space part in a factory was found to vary from day to day. In a sample study, the following information was obtained:								
	Days Mon Tue Wed Thu Fri Sat								
12 A	No. of parts demanded112411251110112011251116								
12 A	Use chi-square to test the hypothesis that number of parts demanded does not $1 + 1 + 1 + 50$ (1 + 1 + 5) (1 + 5)	[10]	CO4						
	depend on the day of the week at 5% level of significance. (Given $\chi^2_{5,0.05} = 11.07$)								
	OR								

Ten students o	ot the	follos	ving ne	ercentao	e of m	arks in	Econor	nics an	d Stati	stics		
Roll No.	-		• •	4	5	6	7	8	9	10		
Marks in Eco.	: 78	36	98	25	75	82	90	62	65	39		
Marks in Stats	: 84	51	91	60	68	62	86	58	53	47		
Calculate the c	oeffic	ient of	correla	tion.								
Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y dz dx + z dx dy)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.												
OR									[10]	CO3		
Apply Green's theorem to evaluate $\int_C \left[(2x^2 - y^2) dx + (x^2 + y^2) dy \right]$ where C is the												
Apply Green's theorem to evaluate $\int_C \lfloor (2x - y) dx + (x + y) dy \rfloor$ where C is the boundary of the area enclosed by the X -axis and the upper half of the circle $x^2 + y^2 = a^2$.												
	Roll No. Marks in Eco. Marks in Stats. Calculate the c Use divergence where <i>S</i> is the Apply Green's boundary of t	Roll No.: 1Marks in Eco.: 78Marks in Stats.:84Calculate the coeffic:Use divergence theorwhere S is the portionApply Green's theorboundary of the are	Roll No.: 12Marks in Eco.: 7836Marks in Stats.:8451Calculate the coefficient ofUse divergence theorem to where S is the portion of the Apply Green's theorem to boundary of the area enclined	Roll No.: 123Marks in Eco.: 783698Marks in Stats.:845191Calculate the coefficient of correlaUse divergence theorem to evaluatwhere S is the portion of the planeApply Green's theorem to evaluatboundary of the area enclosed by	Roll No. : 1 2 3 4 Marks in Eco. : 78 36 98 25 Marks in Stats.: 84 51 91 60 Calculate the coefficient of correlation. Use divergence theorem to evaluate the in where S is the portion of the plane $x + 2y$ O Apply Green's theorem to evaluate $\int_C [(2 + 2y)] dy$	Roll No. : 1 2 3 4 5 Marks in Eco. : 78 36 98 25 75 Marks in Stats.: 84 51 91 60 68 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral \int where S is the portion of the plane $x + 2y + 3z =$ OR Apply Green's theorem to evaluate $\int_C [(2x^2 - y^2) + y^2]$ boundary of the area enclosed by the X -axis	Roll No. : 1 2 3 4 5 6 Marks in Eco. : 78 36 98 25 75 82 Marks in Stats.: 84 51 91 60 68 62 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy)$ where S is the portion of the plane $x + 2y + 3z = 6$ which OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x + y)]$ boundary of the area enclosed by the X -axis and the	Roll No. : 1 2 3 4 5 6 7 Marks in Eco. : 78 36 98 25 75 82 90 Marks in Stats.: 84 51 91 60 68 62 86 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x^2 + y^2))$ boundary of the area enclosed by the <i>X</i> -axis and the upper	Roll No. : 1 2 3 4 5 6 7 8 Marks in Eco. : 78 36 98 25 75 82 90 62 Marks in Stats.: 84 51 91 60 68 62 86 58 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y dz dx + y)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in the first OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x^2 + y^2) dy]$ where boundary of the area enclosed by the <i>X</i> -axis and the upper half of	Roll No. : 1 2 3 4 5 6 7 8 9 Marks in Eco. : 78 36 98 25 75 82 90 62 65 Marks in Stats.: 84 51 91 60 68 62 86 58 53 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y dz dx + z dx dy dy)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octain OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x^2 + y^2) dy]$ where <i>C</i> is boundary of the area enclosed by the <i>X</i> -axis and the upper half of the first of the fi	Marks in Eco. : 78 36 98 25 75 82 90 62 65 39 Marks in Stats.: 84 51 91 60 68 62 86 58 53 47 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y dz dx + z dx dy)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant. OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x^2 + y^2) dy]$ where <i>C</i> is the boundary of the area enclosed by the <i>X</i> -axis and the upper half of the circle	Roll No. : 1 2 3 4 5 6 7 8 9 10 Marks in Eco. : 78 36 98 25 75 82 90 62 65 39 Marks in Stats.: 84 51 91 60 68 62 86 58 53 47 Calculate the coefficient of correlation. Use divergence theorem to evaluate the integral $\iint_{S} (x dy dz + y dz dx + z dx dy)$ where <i>S</i> is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant. OR Apply Green's theorem to evaluate $\int_{C} [(2x^2 - y^2) dx + (x^2 + y^2) dy]$ where <i>C</i> is the boundary of the area enclosed by the <i>X</i> -axis and the upper half of the circle

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2.	Find the Laplace transform $e^t t^{-\frac{1}{2}}$.	[4]	CO2						
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	SECTION B (Q6-Q9 are compulsory and Q10 has internal choice)								
6.	State convolution theorem and hence evaluate $L^{-1}\left(\frac{1}{s(s+1)(s+2)}\right)$.	[8]	CO2						

7.	Solve by rem	noval of	f first oi	der der	ivative	$\frac{d^2y}{dx^2} - 4$	$-x\frac{dy}{dx} + ($	$(4x^2-1)^2$	y = -3	$e^{x^2}\sin 2x$.	[8]	CO1
8.	Using Laplace transform, solve the differential equation $x \frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$, if $y(0) = 2, y'(0) = 0$.										[8]	CO2
9.	An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets.											CO4
10	Apply Stoke's theorem to evaluate $\iint_{S} (\nabla \times F) \cdot \hat{n} ds$, where <i>S</i> is the surface $x^{2} + y^{2} + z^{2} = 4$ above the <i>xy</i> - plane and $\vec{F} = (x^{2} + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^{2})\hat{k}$.											601
10.	OR Evaluate $\iint_{S} (yz\hat{i} + zx\hat{j} + xy\hat{k}) \cdot d\vec{s}$, where <i>S</i> is the surface of the sphere $x^{2} + y^{2} + z^{2} = 16$ in the first octant.								[8]	CO3		
			(Q11	is com		SECTIC y and Q	DN C 12 has	interna	l choic	e)		
11	A) Show that the vector field $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational as well as solenoidal. B) In a partially destroyed laboratory record of an analysis correlation data, the									[10]	CO3	
11.	following results only are legible: Variance of $x=9$, Regression equations are $8x-10y+66=0$; $40x-18y=214$. Find the coefficient of correlation between x and y.							[10]	CO4			
	The marks secured by recruits in the selection test (X) and in the proficiency test (Y) are given below:											
12 A	S. No.: X :	1 10	2 15	3 12	4 17	5 13	6 16	7 24	8 14	9 22		CO4
	Y : Calculate the	30	42	45	46	33	34	40	35	39	[10]	

	OR		
	A survey of 800 families having four children is as follows:		
	No. of male births: $0 1 2 3 4$		
	No. of female births: 4 3 2 1 0		
	No. of families: 32 178 290 236 64		
	Test whether the data is consistent with the hypothesis that the binomial law holds	5	
	and the chance of male birth is equal to that of female birth. ($\chi^2_{4,0.05} = 9.49$).		
	Use divergence theorem to evaluate the integral $\iint_{S} \left(a^{2}x^{2} + b^{2}y^{2} + c^{2}z^{2} \right)^{1/2} ds$		
	where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$.		
12 B	OR		CO3
	Using Green's theorem to evaluate $\int_C \left[(y - \sin x) dx + \cos x dy \right]$ where C is the	;	
	triangle formed by $y = 0, x = \frac{\pi}{2}, y = \frac{2}{\pi}x$.		