Chapter 5

Mitigation of ISI by Equalization Techniques

5.1 INTRODUCTION

Apart from the better receiver and transmitter technology, the OFDM/A communication requires better signal processing techniques to improve the link performance. One such signal processing technique is equalization which is used to compensate the Inter Symbol Interference (ISI) created by multipath transmission within time dispersive channels. An equalizer in the receiver compensates for the average range of expected channel amplitude and the delay characteristics. In other words, an equalizer in the receiver acts as a filter whose impulse response is the inverse of the channel response.

This chapter is organized as follows: section 5.2 gives the mathematical framework of the equalizer followed by the section 5.3 gives overview of zero forcing equalizer and a generic adaptive equalizer in section 5.4 gives the implementation of algorithms for adaptive equalization and section 5.5 shows the comparison between equalization techniques. Section 5.6 summarizes the chapter.

5.2 MATHEMATICAL FRAMEWORK OF EQUALIZER

In the high speed data transmission ISI has been identified as one of the major obstacle which should be removed. Frequency selective fading in which the coherence bandwidth is less than the signal bandwidth, the modulation pulses are spread in time causing ISI.As the mobile fading channels are random and varying continuously with time, an equalizer is expected to track the time varying characteristics of the channel and should be adaptive. Training mode and Tracking mode are the two phases of operation of an Adaptive Equalizer [80]. A pseudo-random binary signal of prescribed bit pattern is generated and thus uses a recursive algorithm to evaluate the channel and estimate the filter coefficients in the training mode. The adaptive algorithms of the equalizer track the changing channel and continuously change the filter characteristics with time in the tracking mode.

The signal at the input of receiver is given as

$$x(t) = m(t) * h(t) + n_b(t) \tag{5.1}$$

Where *m(t)* is the transmitted signal, $n_b(t)$ is the baseband noise and *h(t)* is the combined impulse response of the transmitter, channel and the RF/IF section of the receiver.

If the impulse response of the equalizer is $h_e(t)$, then the output of the equalizer is

$$\begin{aligned}\hat{x}(t) &= m(t) * h(t) * h_e(t) + n_b(t) * h_e(t) \\ &= m(t) * g(t) + n_b(t) * h_e(t)\end{aligned} \tag{5.2}$$

However, the original data source *m(t)* should be the desired output of the equalizer. If $n_b(t)$ is assumed as zero then $\hat{x}(t) = m(t)$ provided

$$g(t) = h(t) * he(t) = \delta(t) \tag{5.3}$$

The main goal of any equalization process is to satisfy the equation optimally i.e. in the frequency domain convolution is simply multiplication and can be written as

$$H(f) \times H_e(f) = 1 \tag{5.4}$$

The equation 5.4 actually tells that the equalizer is actually the inverse filter of the channel. When frequency selective channel is considered the equalizer enhances the frequency components with small amplitudes and attenuates the strong frequencies in the received frequency spectrum. This provides a flat composite received frequency response and linear phase response.

5.3 ZERO FORCING EQUALIZER

In a zero forcing equalizer, the coefficients of the equalizer are chosen in such a way that the impulse response of the samples of channel and the equalizer are forced to zero. The frequency response of the equalizer $H_e(f)$ will be periodic with a period equal to symbol rate 1/T when each of the delay elements provide a time delay T which is equal to symbol duration. Nyquist criterion must be satisfied for the combined response of the channel with equalizer.

$$H_{ch}(f)H_e(f) = 1, \left|f\right| < \frac{1}{2T} \tag{5.5}$$

Where $H_{ch}(f)$ is the folded frequency response of the channel. Hence the equation 5.5 shows that the infinite length zero-forcing equalizer is simply an inverse filter which inverts the folded frequency response of the channel. The only disadvantage with the zero forcing equalizer is that when the folded channel spectrum is subjected to high attenuation the $H_{ch}(f)$ may excessively amplify the noise. The usual equalizer model follows the time varying or adaptive structure equalizer.

5.3.1 A Generic adaptive equalizer

The adaptive equalizer has N delay elements, N+1 taps and N+1 tunable complex multipliers called weights. This filter is called transversal filter. These weights are updated continuously by an adaptive algorithm [81]. A generic adaptive equalizer is shown in Fig.5.1. The adaptive algorithm is controlled by the error signal. The error signal is derived by comparing the output of the equalizer, with some signal which is replica of transmitted signal. The Minimum Mean Square Error (MMSE) can be calculated by equating the differential Mean Square Error (MSE) to zero [82] which is also known as weiner solution.

Since the main task of an adaptive equalizer is to compensate for unknown and time varying channel, a tracking loop system i.e. specific algorithm is needed to update the coefficients and track the channel variations.

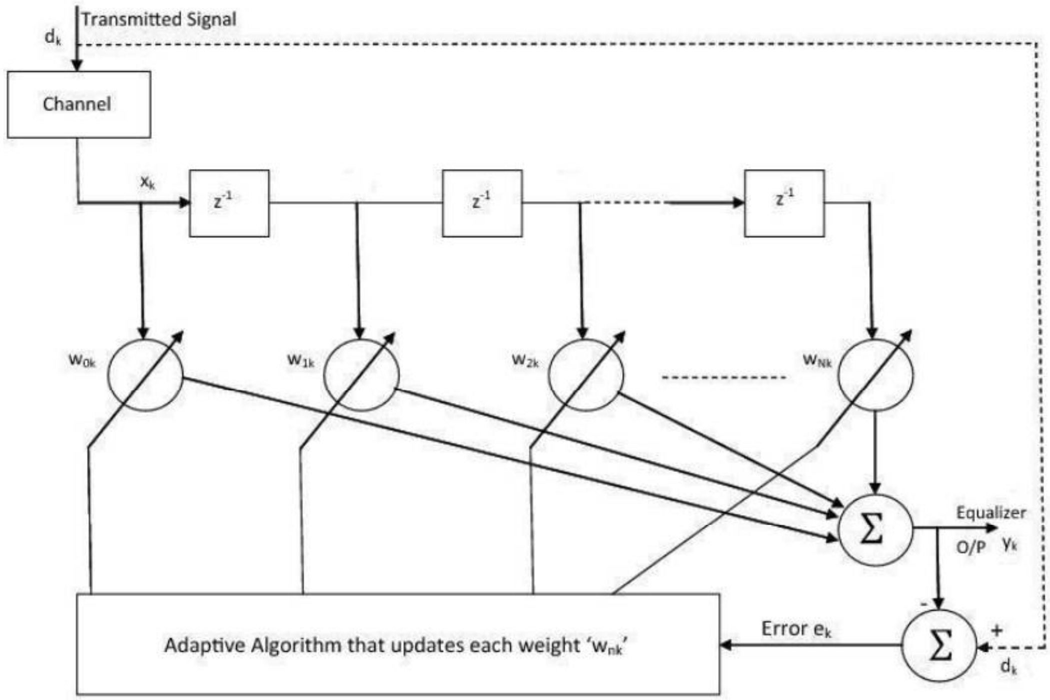

Fig.5.1. A generic adaptive equalizer

Rate of convergence, misadjustment of mean square error, computational complexity and numerical properties like round-off noise are the factors which determine the algorithm's performance.

5.4 EQUALIZER ALGORITHM'S

5.4.1 Least Mean Square (LMS) Algorithm

LMS algorithm is the simplest algorithm for the reduction of the MSE between the actual equalizer output and the desired equalizer output [83]. In the LMS algorithm the system error, MSE and the optimal weiner solution will remain same as adaptive equalizer. It requires 2N+1 operations per iteration. The filter weights will be updated by the update equation. LMS is computed by the equation 5.6.

$$w_k(n+1) = w_k(n) + \mu e_k(n) x(n-k) \tag{5.6}$$

Where the variable n denote the sequence of iteration, subscript k denotes the kth delay stage and μ is the step size by the convergence rate and stability of the algorithm is controlled. x_k and w_k are the input to the equalizer and tap coefficient vector. e_k is the error signal.

The adaptive equalizer becomes unstable if the input signal has time dispersion greater than the propagation delay. In that case the equalizer will be unable to reduce distortion. The only parameter step size controls the adaption rate and hence the convergence rate of LMS algorithm is slow. In the LMS algorithm the value of μ is chosen in the range of

$$0 < \mu < \frac{2}{\sum_{i=1}^{N} \lambda_i} \tag{5.7}$$

Where, λ_i is the i-th eigenvalue of the covariance matrix.

The simulation structure of the LMS equalizer implementation is shown in the Fig.5.2.which is used to calculate the BER.

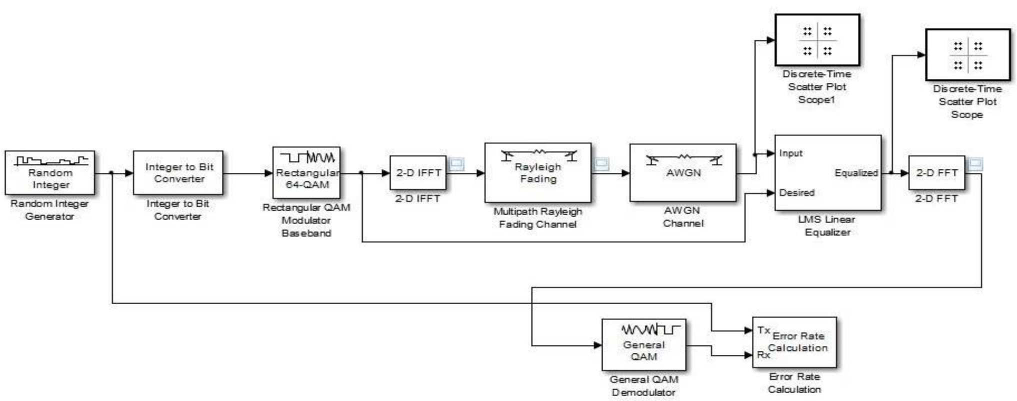

Fig.5.2. BER Analysis of LMS Equalizer for QAM based OFDM/A System

5.4.2 Recursive Least Square (RLS) Algorithm

The RLS algorithm equalizer block uses a linear equalizer and the RLS algorithm to equalize a linearly modulated baseband signal through a dispersive channel [84]. Unlike LMS equalizer algorithm whose aim is to minimize the MSE, RLS equalizer algorithm finds the coefficients that minimize a weighted linear least square cost function relating to input function. The RLS algorithm updates the weights once per symbol during the simulation. When set the Number of samples per symbol parameter is 1, then the block implements a symbol-spaced which is called as T-spaced equalizer and updates the filter weights once for each symbol. If the Number of samples per symbol parameter is set to a value greater than 1, the block updates the weights once every Nth sample, which is called T/N-spaced equalizer or fractionally spaced equalizer.

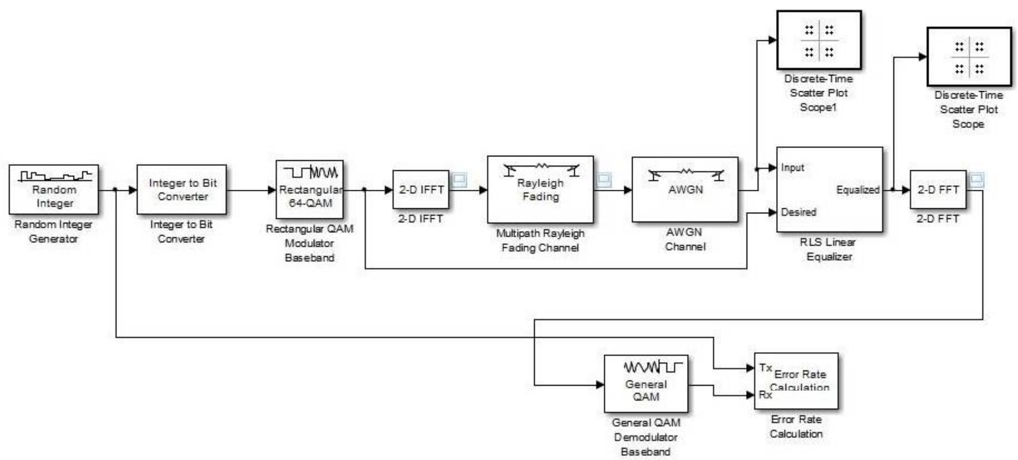

Fig.5.3. BER Analysis of RLS Equalizer for QAM based OFDM/A System

The Simulink simulation structure of the RLS linear equalizer is shown in the Fig.5.3. The BER analysis has been carried out for the equalizer which uses 64-QAM digital modulation and demodulation technique and also implements the 2D-FFT algorithm.

5.4.3 Constant Modulus Algorithm (CMA)

The CMA algorithm uses equalizer block and the algorithm to equalize the linearly modulated baseband signal through a dispersive channel. The equalizer block uses CMA to update the weights. Symbol-spaced equalizer is implemented if the number of samples per symbol parameter is 1, otherwise a fractionally spaced equalizer will be implemented.

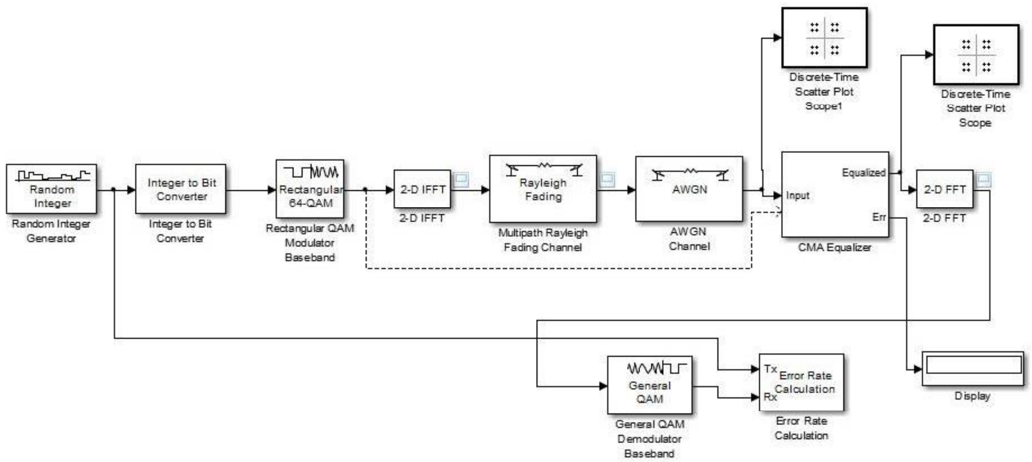

Fig.5.4. BER Analysis of CMA Equalizer for QAM based OFDM/A System

5.5 BER CALCULATIONS

The comparison analysis has been carried between the LMS equalizer, RLS equalizer and the CMA equalizer with different modulation schemes like BPSK, QPSK and QAM techniques. The BER was calculated the same has been shown in the Table.5.1.and the plotting of BER is shown in the Fig.5.5.

Table 5.1 BER Results of LMS, RLS and CMA Equalizer for OFDM/A System

Equalization	BER of OFDM/A based BPSK system	BER of OFDM/A based QPSK system	BER of OFDM/A based QAM system
LMS	0.845	0.909	0.874
CMA	0.818	0.939	0.874
RLS	0.848	0.878	0.938

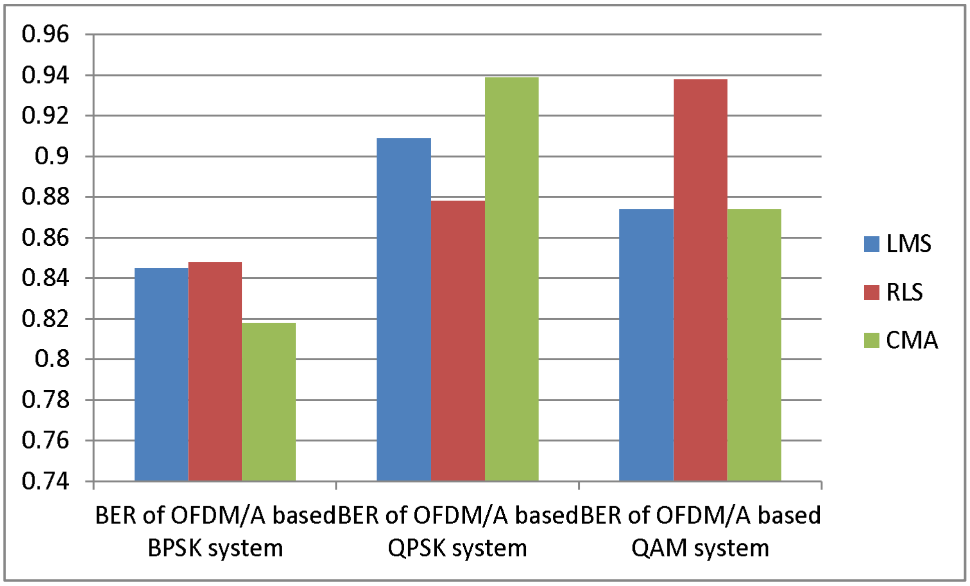

Fig.5.5. BER Analysis of LMS, RLS and CMA Equalizer for OFDM/A System

5.6 SUMMARY

In this chapter starting with the equalization concepts to mitigate the ISI which is caused due to multi path propagation has been studied and then followed by the design equation of the equalizer has been derived which tells that the equalizer is actually the inverse filter of the channel. Zero forcing equalizer has been studied in the next section and the channel response has been derived. A generic adaptive equalizer along with its structure has been studied in the next section which gives the equation for the MSE and the MMSE which should be minimized followed by the next section deals with the various equalizer algorithms like LMS equalizer, RLS equalizer and CMA equalizer algorithms and the implementation of the same was done and BER was calculated for the above algorithms.