

Name: Enrolment No:			
UPES End Semester Examination, May 2025 Course: Integral Equations & Calculus of Variations Program: B.Sc. Physics (R) Course Code: MATH 4019 Semester: VIII Time: 03 hrs. Max. Marks: 100 Instructions: All questions are compulsory.			
SECTION A (5Qx4M=20Marks)			
S. No.		Marks	CO
Q1	Suppose $f: [0,1) \rightarrow [1, \infty)$ is such that $f(x) = 1 + \int_0^x (f(t))^2 dt$. Determine $f(x)$.	4	CO1
Q2	Determine the cardinality of set S consisting of all the solutions of the integral equation: $y(x) = e^x + \int_0^1 2xy(t)dt$	4	CO1
Q3	The integral equation: $\phi(x) = \lambda \int_0^1 e^{x+t} \cdot \phi(t) dt$ has a non-trivial solution for some λ . Find such value(s) of λ .	4	CO2
Q4	Find the resolvent kernel for the Volterra integral equation: $y(x) = x + \lambda \int_a^x y(t) dt$	4	CO2
Q5	Find the set $S = \left\{ y \left(\left(2n + \frac{1}{2} \right) \pi \right) \mid y(x) \text{ is extremal} \right\}$ where the variational problem is $I[y(x)] = \int_0^{2\pi} (y^2 - y'^2)dx$; $y(0) = 1, y(2\pi) = 1$.	4	CO3
SECTION B (4Qx10M= 40 Marks)			
Q 6	Use Laplace transform to determine $y(1)$ from the convolution type integral equation:	10	CO1

	$y(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2] y(t) dt$		
Q7	Use successive approximation to solve the Fredholm equation: $u(x) = 1 + \int_0^1 x \cdot u(t) dt$ with $u_0(x) = 1$ as initial approximation.	10	CO2
Q8	Determine the smooth function $y(x)$ satisfying $y(0) = y(1) = 1$ that minimizes J where $J[y(x)] = \int_0^1 \left(y'^2 + \frac{4y^2}{x^2} \right) x dx$.	10	CO3
Q9	If $y_e(x)$ is the extremal of the functional: $J[y(x)] = \int_0^1 (y'^2(x) + 2y(x)) dx$ subject to $y(0) = 0, y(1) = 1$. Find $\inf J[y_e(x)]$.	10	CO3
SECTION-C (2Qx20M=40 Marks)			
Q10	State the Isoperimetric problem. Use calculus of variations to find the shortest curve in the first quadrant joining points $P(0,0)$ and $Q(1,0)$ that has area equal to 1 square units beneath it.	20	CO4
Q11	(a) Show that the integral equation: $y(x) = f(x) + \lambda \int_0^1 \sin(x+t) y(t) dt$ possesses no solution if $f(x) = x$ but infinitely many solutions if $f(x) = 1$. OR Determine the eigenvalues and eigenfunctions of the integral equation: $y(x) = \lambda \int_0^1 \min[(1-x)t, (1-t)x] y(t) dt$ where $0 < x < 1, 0 < t < 1$.	20	CO2