Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Course: B.Sc. Mathematics **Semester: VI**

Program: Riemann Integration and Series of Functions **Time :03 hrs.**

Course Code: MATH3044P **Max. Marks: 100**

Instructions: Answer all questions.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Given $\int_0^3 x^2 dx = 9$. Find c such that $f(c)$ equals the average value of $f(x) = x^2$ over [0,3].	**04**	**CO1**
Q 2	Show that the sequence $\{f_n\}$ where $f_n = \frac{1}{x+n}$ is uniformly convergent on any interval [0,b], b > 0.	**04**	**CO2**
Q 3	Define pointwise and uniform convergence of a series $\sum f_n$.	**04**	**CO4**
Q 4	Find the radius of convergence of the series $$\frac{1}{2}x + \frac{1.3}{2.5}\ x^2 + \frac{1.3.5}{2.5.8}x^3 + \cdots$$	**04**	**CO3**
Q 5	Define open and closed spheres in a metric space (X, d).	**04**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Prove that $\int_0^\infty \frac{x \tan^{-1} x}{\sqrt[3]{1+x^4}} dx$ is divergent	**10**	**CO2**
Q 7	If a sequence $\{f_n\}$ converges uniformly to f on [a, b], and each function f_n is integrable, then f is integrable on [a, b] and the sequence $\left\{\int_a^x f_n dt\right\}$ converges uniformly to $\int_a^x f\ dt$ on [a, b], i.e., $$\int_a^x f\ dt = \lim_{n\to\infty} \int_a^x f_n dt, \forall\ x \in [a, b].$$	**10**	**CO2**
Q 8	Show that the series $\sum f_n$ whose sum function is given by $S_n(x) = nxe^{-nx^2}$ is pointwise and not uniformly convergent on any interval [0, k], k > 0.	**10**	**CO4**

Q 9	Let (X,d) be any metric space. Show that the function d_1 defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}, \forall\, x,y \in X$, is a metric on X. **(OR)** Show that the set $\mathbb{R}^n$ of all ordered n-tuples with function d defined by $$d(x,y) = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}$$ $\forall\, x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, \dots, y_n) \in \mathbb{R}^n$, is a metric space.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(a) If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$, then prove that $f \in R(\alpha_1 + \alpha_2)$ and $\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$. (b) If $f \in R(\alpha)$ and c is a positive constant, then prove that $f \in R(c\alpha)$ and $\int_a^b f\, d(c\alpha) = c \int_a^b f\, d(\alpha)$.	**10+10**	**CO1**
Q 11	State and prove Dini's Theorem on uniform convergence of a sequence $\{f_n\}$. **(OR)** State and prove Cantor's Intersection Theorem.	**20**	**CO4**