Name:

Enrolment No:

UPES

End Semester Examination, May 2025

Course: Time Series and Forecasting Methods **Semester: VI**

Program: B.Sc. (Hons.) Mathematics **Time: 03 hrs.**

Course Code: MATH 3037P **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Calculate the difference between four-year and three-year moving average forecast for 2025 Year: 2020, 2021, 2022, 2023, 2024 Demand: 55, 60, 58, 65, 72	4	CO3
Q 2	Find the semi average for the following data: 1981: 132 1982: 135 1983: 141 1984: 144 1985: 136 1986: 138 1987: 117 1988: 144 1989: 151 1990: 140	4	CO1
Q 3	Show that difference of two independent Poisson processes is not a Poisson process.	4	CO2
Q 4	The joint pdf of (X, Y) is given by $f_{XY}(x,y) = \begin{cases} ke^{-(x+2y)} & x > 0, y > 0 \\ 0 & \text{Otherwise} \end{cases}$ where k is a constant. Find $P(X > 1, Y < 1)$, $P(X < Y)$ and $P(X \leq 2)$.	4	CO4
Q 5	Discuss the difference between AR process and IMA process.	4	CO3

SECTION B
(4Qx10M= 40 Marks)

Q 6	Define power spectral density function. Also show that the power spectral density function is an even function.	**10**	**CO1**
Q 7	Consider a random process $X(t)$ defined by $X(t) = Ycoswt \qquad t \geq 0$ where w is a constant and Y is a uniform random variable over $(0, 1)$. i) Describe $X(t)$. ii) Sketch a few typical sample functions of $X(t)$. iii)Find $E[X(t)]$.	**10**	**CO2**
Q 8	Consider the ARMA (1, 2) process defined by the equation $X_t = 0.5\, X_{t-1} + Z_t - 0.3Z_{t-1} + 1.2\, Z_{t-2}, \quad \{Z_t\} \sim WN(0, \sigma^2)$. Test the causality and invertibility of the process.	**10**	**CO2**
Q 9	Compute the trend and short-term error by the method of moving averages assuming a 3-yearly cycle is present in the following series. Year: 1987 1988 1989 1990 1991 1992 Production(in tons): 21 22 23 25 24 22 Year: 1993 1994 1995 1996 1997 1998 Production(in tons): 25 26 27 26 23 26 **OR** Consider the following auto regressive model of order 2, $X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + Z_t$. For the following data set X_t: 5.81, 4.72, 6.85, 4.02, 3.66, 1.99, -1.13, -3.82, -5.08, -4.42. Find the values of $\rho(1), \rho(2), \gamma(1), \gamma(2)$.(Symbol having their usual meaning.)	**10**	**CO3**

SECTION-C
(2Qx20M=40 Marks)

Q 10	Let $Z_1, Z_2, \ldots$ be independent identically distributed random variables with $P(Z_n = 1) = p$ and $P(Z_n = -1) = q = 1 - p$ for all n. Let $X_n = \sum_{i=1}^{n} Z_i,\ n = 1, 2, \ldots\ldots$ and $X_0 = 0$. a) Find the probability that $X_n = -2$ after four steps. b) Verify the result of part (a) by enumerating all possible sample sequences that lead to the value $X_n = -2$ after four steps.	**20**	**CO1**
Q 11	The following table gives the sales figures for a hundred units of a product.	**20**	**CO4**

	Quarters			
Year	**I**	**II**	**III**	**IV**
2012	15	20	18	17
2013	17	26	25	72
2014	20	29	27	24
2015	27	38	34	31
2016	40	46	43	41

Use the least square method to form equation and compute the demand for the product for the quarter wise using ratio to trend method.

OR

The following data give the average quarterly prices of a commodity for five years. Calculate seasonal indices by the method of link relatives.

Year/Quaters	Quaterly output for 5 years			
	I	II	III	IV
2011	30	26	22	31
2012	35	28	22	36
2013	31	29	28	32
2014	31	31	25	35
2015	34	36	26	33