Name:

Enrolment No:

UPES

End Semester Examination, May 2025

Course: Mathematical Methods **Semester: VI**

Program: B.Sc. Mathematics **Time : 03 hrs.**

Course Code: MATH3033 **Max. Marks: 100**

Instructions: 1. This question paper contains 11 questions.
2. Attempt all questions from Section A (each carrying 4 marks).
3. Attempt all questions from Section B (each carrying 10 marks). Question 9 has an internal choice.
4. Attempt all questions from Section C (each carrying 20 marks). Question 11 has an internal choice.
5. Calculator is allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Derive the Laplace transform of the function, $f(t) = t$.	**4**	**CO1**
Q 2	Write the Second Shifting Theorem for Laplace transform and find the Laplace transform of $f(t) = \begin{cases} t, & 0 \le t < 2 \\ t+1, & t \ge 2 \end{cases}$.	**4**	**CO1**
Q 3	Compute two iterations for a root of $x^3 + 2x - 7 = 0$ using the Regula-Falsi method, in the interval $(1, 2)$.	**4**	**CO2**
Q 4	Write the Newton's forward interpolation formula for a function $y = f(x)$, if it takes the values y_0, y_1, y_2 corresponding to $x = x_0, x_1, x_2$.	**4**	**CO3**
Q 5	Given x: 5, 10, 15, 20 y: -12, 3, 10, 16 Show that $\Delta^2 y_1 = \nabla^2 y_3$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the value of $e^{12.2}$ using Bessel's interpolation formula if $x \in [10,15]$ with $h = 1$.	**10**	**CO3**

Q 7	Solve the following system using two iterations of the Gauss-Siedel method. $4x - y + z = 4$ $x + y - 5z = 7$ $2x + 6y + z = -1$	**10**	**CO2**
Q 8	A curve passes through the points $(0, 18), (1, 10), (2, -18)$ and $(3, 90)$. Find the slope of the curve at $x = 3$.	**10**	**CO4**
Q 9	Solve the following differential equation using Laplace transform: $y'' = e^t, y(0) = 1,\ y'(0) = 0.$ **OR** Compute $L^{-1}\left(\frac{1}{(s-2)(s+1)^2}\right)$.	**10** **10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Evaluate $\int_0^6 \frac{1}{5+3x}\,dx$ using a) Simpson's 1/3rd rule b) Simpson's 3/8th rule by taking n=6. Also, compare these values with the actual value of the integral.	**20**	**CO4**
Q 11	Find the Fourier sine transform of $f(x) = e^{-x}$ and hence show that $\int_0^\infty \frac{\alpha \sin \alpha x}{1+\alpha^2}\,d\alpha = e^{-x}\frac{\pi}{2}$. **OR** Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.	**20**	**CO1**