Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Course: Advanced Solid Mechanics **Semester: VI**

Program: Aerospace Engineering **Time: 03 hrs.**

Course Code: MECH 3057P **Max. Marks: 100**

Instructions: Attempts all questions. Take the appropriate coordinate system whenever required.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	The tensor stress transformation equation is expressed as $\sigma'_{mn} = l_{mi}l_{nj}\sigma_{ij}$, where $i,j,m,n = 1,2,3$. In which, σ_{ij} is the state of stress tensor and σ_{mn} is the stress tensor in transformed coordinate in plane whose direction cosines are l_{ij} with respect to principle stresses. According to this tensor equation write the expression for σ'_{13}.	**4**	**CO1**
Q 2	A rectangular element used in a plane stress analysis has corners whose coordinates (in metres), referred to an Oxy axes system, are 1(−2, −1), 2(2, −1), 3(2, 1) and 4(−2, 1); the displacement field (also in metres) were expressed as $u = 5y^2 - 6xy + 10$, and v= $x^2 + 8y + 3xy - 9$. Determine the normal strain and rotation at the centre of the element.	**4**	**CO1**
Q 3	A thin-walled cylindrical pressure vessel 8 m long has closed ends, a wall thickness of 4 mm and an inner diameter of 2 m. if the vessel is filled with air to a pressure of 1.5 MPa. Determine the change in length of cylinder. Take E = 200 GPa and $\nu = 0.30$.	**4**	**CO1**
Q 4	Derive the relation between E, K and ν, where E is Modulus of elasticity, K is Bulk's Modulus and is ν Poisson's ratio.	**4**	**CO1**
Q 5	If $\sigma_1 = \sigma_2 = -200$ MPa and $\sigma_3 = 400$ MPa. Determine the σ and τ on plane with direction cosines $(0, 1/2, \sqrt{3}/2)$.	**4**	**CO1**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Derive the governing equation of stress distribution and the displacement field in thick cylinder. Compare the radial and hoop stresses for a cylinder which is subjected to only internal pressure not external pressure and vice-versa, with suitable plot.	**10**	**CO2**

Q 7	Derive the differential equations of equilibrium for the plane stress condition in rectangular coordinate system for a stressed element.	**10**	**CO2**
Q 8	Given that the body force is negligible. Investigate if the following displacement field can be a solution of a static plane stress problem $u = c_1(x^2 - y^2) - c_2 y + c_3$ and $v = 2a_1 xy + a_4$ where the a_i are constants.	**10**	**CO3**
Q 9	Given a state of stresses at a point with respect to a convenient coordinate system (x, y, z) be σ_x = 500 MPa, $\sigma_y = -40$ MPa, $\sigma_z = 30$ MPa, $\tau_{xy} = 20$ MPa, $\tau_{xz} = \tau_{yz} = 10$ MPa. Determine the principal normal stresses and the direction cosine of the principal planes. **Or,** Given a state of strains at a point with respect to a convenient coordinate system (x, y, z) be $\varepsilon_x = -100\ \mu\varepsilon$, $\varepsilon_y = 300\ \mu\varepsilon$, $\varepsilon_z = -400\ \mu\varepsilon$, $\gamma_{xy} = -280\ \mu\varepsilon$, $\gamma_{yz} = -86\ \mu\varepsilon$ and $\gamma_{xz} = -100\ \mu\varepsilon$. Determine the principal normal strains and the direction cosine of the principal normal strains.	**10**	**CO3**
SECTION-C (2Qx20M=40 Marks)			
Q 10	Explain the governing equations for the stresses produced in non-circular shaft with torsion. Define the Prandtl's stress function for any non-circular shaft and relate it with the applied twisting moment on the shaft. Drive the expression of Prandtl's stress function for the rectangular cross-section of shaft subjected to twisting moment. Show the stress distribution for the rectangular cross-section of shaft.	**20**	**CO4**
Q 11	A thick cylinder has inner and outer diameters as 150 and 200 mm respectively. It is subjected to an external pressure of 12 MPa. Determine the value of internal pressure which can be applied if the maximum stress is not to exceed 40 MPa. Draw the curves showing the variation of hoop and radial stresses through the material of the cylinder. Or, A disc of uniform thickness and of 400 mm diameter rotates at 7500 rpm. Determine the maximum stress developed in the disc. If a hole of 100 mm diameter is made at the center of the disc, determine the maximum values of radial and hoop stresses. Density of material of the disc is 7500 kg/m^3 and $\nu = 0.25$.	**20**	**CO3**