Name: **Enrolment No:**	

UPES
End Semester Examination, May 2025

Programme Name : B.Sc. (H) Mathematics By Research **Semester : IV**
Course Name : Complex Analysis **Time : 03 hrs**
Course Code : MATH 2057 **Max. Marks: 100**
Nos. of page(s) : 02

Instructions: All questions are compulsory. Use of calculator is not allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Determine the biggest subset S of the set of complex numbers $\mathbb{C}$ on which the complex function $f(z) = e^{z^2} + (1+i)\bar{z}$ is differentiable.	**4**	**CO1**
Q2	Suppose $u(x,y)$ and $v(x,y)$ are both harmonic functions on domain U. Is $f = u + iv$ analytic on U? Justify your answer.	**4**	**CO1**
Q3	Evaluate the contour integral $$\oint_C z\, d\bar{z}$$ where contour C is the unit circle $\lvert z-a \rvert = r$ oriented counterclockwise.	**4**	**CO2**
Q4	If $k(>0)$ is an integer then find the radius of convergence of power series $$\sum_{n=1}^{\infty} n\ln(n+1)\ z^{kn}$$	**4**	**CO2**
Q5	Find the residue of the function $f(z) = \frac{e^{\frac{1}{z}}}{\sinh z}$ at the singularity $z = 0$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Discuss the existence of the limit $\lim_{z\to 0} e^{\left(1+\frac{1}{z^4}\right)}$ using suitable paths passing through the point $z = 0$.	**10**	**CO1**
Q7	Suppose $f = u + iv$ is entire such that $u^2 \leq v^2\ \forall z \in \mathbb{C}$. Is $f \equiv$ constant? Prove or give counterexample to discard the statement.	**10**	**CO2**

Q8	Use Laurent series expansion by defining a suitable annular open connected set $r < \|z-1\| < R$ to comment correctly on the nature of singularity for $f(z) = \frac{z}{z^2-1}$ at the point $z = 1$.	**10**	**CO2**
Q9	Suppose C is an arbitrary closed simple curve on complex plane with unknown orientation. Evaluate the integral $$\left\|\frac{1}{2\pi}\right\| \oint_C \frac{\sin z \cos^2 z + z^{2025}}{e^{2023z}} dz$$ **OR** Determine the value of $k \in \mathbb{Z}_{>0}$ so that $$\frac{1}{2\pi i} \oint_C \frac{z^2 - z - k}{z - k} dz = 0$$ where C is an arbitrary closed simple curve enclosing the point $z = k$ on complex plane.	**10**	**CO3**
SECTION-C (2Qx20M=40 Marks)			
Q10	Consider $f(z) = \frac{e^{\frac{1}{z}}}{1-\cos z}$. (i) Determine all the singularities of $f(z)$. (ii) Discuss the behavior of $f(z)$ at the singularity $z = 0$. (iii) If C is the circle $z = e^{i2\theta}$, where $\theta \in [0,2\pi)$ then find the value of $$\oint_C z^2 f(z) dz$$ (iv) Find the order of poles at $z = 2\pi k, k \in \mathbb{Z}\backslash\{0\}$.	**20 [5+5+ 5+5]**	**CO2**
Q11	Use calculus of residues to prove the following $$\int_0^{2\pi} \frac{\sin^2\theta}{a + b\cos\theta} d\theta = \frac{2\pi}{b^2}\left(a - \sqrt{a^2 - b^2}\right), a > b > 0$$ **OR** Find the principal value of the real integral $$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 - x + 2)} dx,$$ by clearly showing how the value of the integral $\int \frac{e^{iz}}{z(z^2-z+2)} dz \longrightarrow 0$ along the semicircular arc in upper half complex plane.	**20**	**CO3**