Name: Enrolment No:	UPES University of Tomorrow

UPES

End Semester Examination, May 2025

Course: **Partial Differential Equations** — **Semester : IV**

Program: **B. Sc. (Hons.) Mathematics by Research** — **Time : 03 hrs.**

Course Code: MATH2054 — **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 6 and 11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1, \log_e 2)$ where the function $f(x,y) = xe^{x^2y}$.	4	CO1
Q 2	Locate the stationary point(s) of the function $$f(x,y) = (x+y)e^{-x^2-y^2}.$$	4	CO1
Q 3	Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(x)$ from the relation given as $$z(x,y) = f(x) + e^y g(x).$$	4	CO2
Q 4	Find the general solution of Lagrange's equation $$yzp + zxq = xy,$$ where p stands for $\frac{\partial z}{\partial x}$ and q stands for $\frac{\partial z}{\partial y}$.	4	CO2
Q 5	Determine the region in the xy-plane in which the following equation is hyperbolic: $$[(x-y)^2 - 1]u_{xx} + 2u_{xy} + [(x-y)^2 - 1]u_{yy} = 0.$$	4	CO3

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Expand the function $f(x,y) = e^{2x-y}$ in a Taylor's series about the point $(x,y) = (0,1)$ upto the quadratic terms. From this series estimate the value of $f(x,y)$ at $(-0.1, 1.1)$. **OR** A tent of a given volume has a square base of side $2a$, has its four-side vertical of length b and is surmounted by a regular pyramid of height h. Find the values of a and b in terms of h such that the canvas required for its construction is minimum.	10	CO1

Q 7	Apply Charpit's method to find the complete solution of the non-linear partial differential equation $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$ (where $p \equiv \frac{\partial z}{\partial x}$ and $q \equiv \frac{\partial z}{\partial y}$.)	**10**	**CO2**
Q 8	Solve the partial differential equation $(D + D' - 1)(D + 2D' - 3)z = 4 + 3x + 6y,$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.	**10**	**CO3**
Q 9	Find the temperature in a laterally insulated bar of $2\ cm$ length whose ends are kept at zero temperature and the initial temperature is $\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**(i)** Find the general solution of the partial differential equation $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y) + e^{3x+y},$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$. **(ii)** Reduce the differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form.	**10+10**	**CO3**
Q 11	The vibration of an elastic string is governed by the partial differential equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$ The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string at any time t. **OR** Solve the partial differential equation: $\frac{\partial^2 y}{\partial t^2} = 4\frac{\partial^2 y}{\partial x^2}$, with the conditions $y(0,t) = y(5,t) = 0; \quad y(x,0) = 0,$ and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3\sin 2\pi x - 2\sin 5\pi x.$	**20**	**CO4**