Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Course: Mathematical Physics III **Semester: IV**
Program: B.Sc. (Hons.) Physics **Time: 03 hrs.**
Course Code: PHYS 3056 **Max. Marks: 100**

Instructions:
1. All questions are compulsory (Q. No. 9 and Q. No. 11 have internal choices).
2. Use of scientific calculators is not allowed during the test.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	State Cauchy's residue theorem.	**4**	**CO1**
Q 2	Define three dimensional Fourier transform along with the inverse formula for the same.	**4**	**CO2**
Q 3	Find the Laplace transform of $f(t) = e^{t^2}$.	**4**	**CO2**
Q 4	Let $z = a + ib$, with a and b real. Find the real part of $\cos(a + ib)$.	**4**	**CO1**
Q 5	State the Dirichlet conditions for the convergence of a Fourier series.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Define $f(z) = e^z$. Let a be a positive real number, and let C be the rectangle with vertices $0, a, a + 2\pi i, 2\pi i$. Explicitly evaluate the integral $\oint f(z)dz$ over C without using Cauchy's theorem and illustrate that the theorem applies in this case.	**10**	**CO3**
Q 7	Understanding the function $f(x) = \cosh x$ defined over the period $[-\pi, \pi]$ expand it into an appropriate Fourier series.	**10**	**CO4**
Q 8	Remembering $f(x)$ and $g(\alpha)$ to be a pair of Fourier transforms, derive that $\frac{df}{dx}$ and $i\alpha g(\alpha)$ are a pair of Fourier transforms as well.	**10**	**CO2**

Q 9	Find the Laplace transform of Delta function $\delta(t-t_0)$, use it to solve $y'' + \omega^2 y = \delta(t-t_0)$ with the boundary conditions $y_0 = y'_0 = 0$. **OR** Use Laplace transform to solve: $y'' + 9y = \cos 2t$ with the boundary conditions $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$.	**3+7 = 10** **10**	**CO3** **CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Solve the equation $$y''(t) + 8y'(t) + 7y(t) = f(t) = \begin{cases} e^{-at}; & t > 0 \\ 0; & t < 0 \end{cases}$$	**20**	**CO4**
Q 11	Define $f(z) = \frac{ze^{iz}}{(z^2+a^2)(z^2+b^2)}$. Find all the singularities of the function in the region $Im\, z \geq 0$ and compute their residues. Hence show that $\int_{-\infty}^{+\infty} \frac{x \sin x}{(x^2+a^2)(x^2+b^2)}\, dx = \frac{\pi}{(b^2-a^2)}(e^{-a} - e^{-b})$ for $0 < a < b$. **OR** Evaluate $\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{e^{zt}}{\sqrt{z+1}} dz$ where a and t are any positive constants.	**10+10** **20**	**CO3**