Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES
End Semester Examination, May 2025

Course: Linear Algebra-II **Semester: II**
Program: B.Sc. (Hons.) Mathematics by Research **Time : 03 hrs.**
Course Code: MATH 1063 **Max. Marks: 100**

Instructions: All questions are compulsory. Use of calculator is not allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let V be a vector space of dimension 7 over $\mathbb{R}$ and let $T: V \to V$ be a linear operator with minimal polynomial $m(t) = (t^2+1)(t+2)^3$. Find all the possible rational canonical forms for T.	**4**	**CO1**
Q 2	Prove that all the eigenvalues of a nilpotent linear transformation are zero.	**4**	**CO1**
Q 3	Let $T: \mathbb{C}^3 \to \mathbb{C}^3$ be defined by $T(x,y,z) = (ix + (2+3i)y, 3x + (3-i)z, (2-5i)y + iz)$ Find the adjoint of T, *i.e.*, $T^*(x,y,z)$.	**4**	**CO2**
Q 4	Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x,y) = (3x+3y,\ x+5y)$. Find the characteristics polynomial and all eigenvalues of T. Is T diagonalizable?	**4**	**CO3**
Q 5	Define normal operators. State Spectral theorem for a finite dimensional inner product space V.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Find the basis $\{\phi_1, \phi_2, \phi_3\}$ that is dual to the following basis of $\mathbb{R}^3$: $\{v_1 = (1,-2,3), v_2 = (1,-1,1), v_3 = (2,-4,7)\}$.	**10**	**CO3**
Q 7	a) Prove that the eigenvalues of a self-adjoint linear operator are all real. b) Show that eigenvectors corresponding to the distinct eigenvalues of a self-adjoint linear operator are orthogonal to each other.	**5+5**	**CO2**

Q 8	Let T_1 and T_2 be two linear operators on an inner product space V and let $k \in F$ be a scalar. Then, prove the following: a) $(T_1 + T_2)^* = T_1^* + T_2^*$ b) $(kT)^* = \bar{k}T^*$	**5+5**	**CO2**
Q 9	State and prove Bessel's inequality for an inner product space $(V(F), <\ ,\ >)$. **OR** Find the Fourier coefficient and the projection of $v = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ along $w = \begin{pmatrix} 1 & 1 \\ 5 & 5 \end{pmatrix}$ in a vector space $V = M(2, \mathbb{R})$ with the inner product defined by $$< A, B > = trace(B^T A)$$ where B^T is the transpose of matrix B.	**10**	**CO4**
SECTION-C (2Qx20M=40 Marks)			
Q 10	a) Let U and W be subspaces of a vector space V of finite dimension. Prove that $$(U + W)^0 = U^0 \cap W^0$$ where X^0 denotes the annihilator of a set X. b) Find a basis of the annihilator W^0 of the subspace W of $\mathbb{R}^4$ spanned by $v_1 = (1,2,-3,4)$ and $v_2 = (0,1,4,-1)$.	**10+10**	**CO3**
Q 11	Suppose $v = (1,3,5,7)$. Find $w \in W$ that minimizes $\|\|v - w\|\|$, where W is the subspace of $\mathbb{R}^4$ spanned by $v_1 = (1,1,1,1)$ and $v_2 = (1,2,3,2)$. **OR** a) Let S be a subset of a vector space V. Show that orthogonal complement of $S, i.e., S^\perp$ is a subspace of V. b) Obtain an orthonormal basis from the standard basis of $P_2(\mathbb{R})$, the space of all real polynomials of degree ≤ 2, the inner product being defined by $$< f(t), g(t) > = \int_{-1}^{1} f(t)g(t)dt$$	**20**	**CO4**