Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Course: Number Theory | **Semester : II**

Program: M.Sc. Mathematics | **Time : 03 hrs.**

Course Code: MATH7026P | **Max. Marks: 100**

Instructions: Answer the following questions as per the instructions.

SECTION A

(5Q x 4M = 20Marks)

S. No.		Marks	CO
Q 1	Evaluate $\left(\frac{-219}{383}\right)$, $\left(\frac{a}{p}\right)$ represents the Legendre symbol.	**4**	**CO3**
Q 2	Let r be the remainder when $1059, 1417, 2312$ are divided by $d > 1$, then find $d - r$.	**4**	**CO1**
Q 3	Justify the following statement. In Pell's equation $x^2 - dy^2 = 1$, d is taken to be square-free.	**4**	**CO4**
Q 4	Find the multiplicative inverse of 29 under (mod 48).	**4**	**CO2**
Q 5	Illustrate the fact that the Diophantine equation $x^n + y^n = z^n$ has the trivial integral solution, provided $xyz = 0$ and $n \geq 3$.	**4**	**CO4**

SECTION B

(4Q x 10M = 40 Marks)

S. No.		Marks	CO
Q 6	Prove that if p is a prime number and $(a, p) = 1$, then the congruences $x^n \equiv a\ (mod\ p)$ has $(n, p-1)$ solutions or no solutions according as $a^{(p-1)/(n,p-1)} \equiv 1\ (mod\ p)$ or not.	**10**	**CO2**
Q 7	Suppose p is an odd prime and integer a be such that $(a, p) = 1$. Consider set $S = \left\{a, 2a, 3a, \dots, \frac{p-1}{2}a\right\}$ having subset $A = \left\{b \in S : b\ (modp) > \left\lfloor\frac{p}{2}\right\rfloor\right\}$ such that $\|A\| = n$, then show that $\left(\frac{a}{p}\right) = (-1)^n$.	**10**	**CO3**
Q 8	Find the general formula for the nth Triangular and Pentagonal number.	**10**	**CO4**

Q 9	Prove that, $\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \ (mod\ p)$, for any odd prime p. Hence find $\left(\frac{5}{11}\right)$. **OR** Let m, n be odd integers, then prove that $\left(\frac{n}{m}\right) = (-1)^{\frac{n-1}{2}.\frac{m-1}{2}}\left(\frac{m}{n}\right)$. Hence evaluate $\left(\frac{3}{11}\right)$.	**10**	**CO3**
SECTION-C **(2Q x 20M = 40 Marks)**			
Q 10	Find all possible solutions of the congruence $x^2 \equiv 79\ (mod\ 91)$.	**20**	**CO3**
Q 11	Let (x, y, z) be a Pythagorean triplet. Then show that (a) x and y are of different integral structures, provided z is odd. (b) For any odd x, there exist $a, b \in \mathbb{N}$ with $a > b, (a, b) = 1$ and $a \not\equiv b\ (mod\ 2)$ satisfying $x = a^2 - b^2,\ y = 2ab,\ z = a^2 + b^2$. **OR** (a) Prove that (x, y) is a solution to the Pell's equation $x^2 - 2y^2 = 1$ if and only if $x + y\sqrt{2} = (3 + 2\sqrt{2})^n$ for some $n \in \mathbb{N}$. (b) Prove that Triangular – Square equation $T_m = S_n$ is shifted to $x^2 - 2y^2 = 1$ under the transformation $x = 2m + 1, y = 2n$ and reversely.	**10 + 10**	**CO4**