Name: Enrolment No:	UPES UNIVERSITY OF TOMORROW

UPES
End Semester Examination, May 2025

Course: **Theory of Partial Differential Equations** **Semester : II**
Program: **M. Sc. Mathematics** **Time : 03 hrs.**
Course Code: MATH7024 **Max. Marks : 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 6 and 11 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Form a partial differential equation by eliminating the arbitrary constants a and b from the following relation $$z(x,y) = axe^y + \frac{1}{2}a^2e^{2y} + b.$$	**4**	**CO1**
Q 2	Find the general solution of Lagrange's equation $$p - q = \log_e(x+y), \text{ (where } p \equiv \frac{\partial z}{\partial x} \text{ and } q \equiv \frac{\partial z}{\partial y}\text{).}$$	**4**	**CO1**
Q 3	Discuss Reducible and irreducible linear partial differential equations with constant coefficients by suitable examples.	**4**	**CO2**
Q 4	Show that $u(x,t) = \sin(\pi x)e^{-4t\pi^2}$, is a solution of the one-dimensional heat equation given by $$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}.$$	**4**	**CO3**
Q 5	Write a short note on the formulation of the one-dimensional wave equation.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Determine the region in the xy-plane where the following equation: $$(1+x)u_{xx} + 2xyu_{xy} - y^2u_{yy} = 0,$$ is elliptic, hyperbolic or parabolic. **OR** Solve the partial differential equation $$(D^2 - DD')z = \cos 2y(\sin x + \cos x)$$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.	**10**	**CO1**

Q 7	Transform the two-dimensional Laplace equation $$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,$$ in polar coordinates.	**10**	**CO2**
Q 8	Reduce the partial differential equation $$\frac{\partial^2 z}{\partial x^2}-(1+y)^2\frac{\partial^2 z}{\partial y^2}=0.\,(y\neq -1)$$ to canonical form.	**10**	**CO3**
Q 9	Derive D'Alembert's solution to the one-dimensional wave equation.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	A laterally insulated bar of length l has its ends A and B maintained at 0^0C and 100^0C respectively, until steady-state conditions prevail. If the temperature at B is suddenly reduced to 0^0C and kept so while that of A is maintained at 0^0C, find the temperature at a distance x from A at any time t.	**20**	**CO3**
Q 11	The vibration of an elastic string is governed by the partial differential equation $$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}.$$ The length of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0)=2(\sin x+\sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string at any time t. **OR** A tightly stretched string with fixed endpoints $x=0$ and $x=\pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity given as $$\left(\frac{\partial y}{\partial t}\right)_{t=0}=0.03\sin x-0.04\sin 3x,$$ then find the displacement $y(x,t)$ at any point of the string at any time t.	**20**	**CO4**