Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2025

Programme Name : M.Sc. Mathematics **Semester : II**
Course Name : Complex Analysis **Time : 03 hrs**
Course Code : MATH 7022 **Max. Marks: 100**
Nos. of page(s) : 02

Instructions: All questions are compulsory. Use of calculator is not allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q1	Does the limit $\lim_{z\to 0} e^{\left(z+\frac{1}{z^4}\right)}$ exist? Justify your answer using suitable paths passing through the point $z=0$.	**4**	**CO1**
Q2	Determine an analytic function $f(z)$ such that $Ref(z)=e^x(x\cos y - y\sin y)$.	**4**	**CO1**
Q3	Evaluate the contour integral $$\oint_C \lvert z\rvert^2 \, \lvert d\bar{z}\rvert$$ where contour C is the unit circle $\lvert z-a\rvert = r$ oriented counterclockwise.	**4**	**CO2**
Q4	If $k(>0)$ is an integer, then find the radius of convergence of power series $$\sum_{n=1}^{\infty}(n+k)\ln(n+1)\ z^{k(n!)}$$	**4**	**CO3**
Q5	Find the points on complex plane where the function $f(z)=z+\frac{1}{3z^3}$ is not conformal. Also determine the magnification factor at the critical points.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the residue of the function $f(z)=\frac{e^{\frac{1}{z}}}{\sinh z}$ at the singularity $z=0$.	**10**	**CO1**
Q7	Suppose $f=u+iv$ is entire such that $u^2\le v^2\ \forall z\in\mathbb{C}$. Is $f\equiv$ constant? Prove or give counterexample to discard the statement.	**10**	**CO2**

Q8	Use Laurent series expansion by defining a suitable annular open connected set $r < \|z-1\| < R$ to comment correctly on the nature of singularity for $f(z) = \frac{z}{z^2-1}$ at the point $z = 1$.	**10**	**CO2**
Q9	Suppose C is an arbitrary closed simple curve on complex plane not passing through $z = 0$ with unknown orientation. Evaluate the integral $$\frac{1}{2\pi i}\oint_C \frac{\sin\frac{1}{z}}{ze^{\frac{1}{z}}}dz$$ **OR** Suppose C is a circle $\|z\| = \frac{1}{2}$ oriented counterclockwise and $f(z) = e^z/g(z)$ where $g(z)$ is a polynomial of degree 10 having simple zeroes that are equally spaced within the interval $[0,9]$ of real axis. If $g(0) = g(9) = 0$, find the value of $$\oint_C f(z)dz$$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(i) Find the image of $\{z \in \mathbb{C} \mid \|z\| < 1, Im(z) \geq 0\}$ under the linear fractional transformation $w(z) = \frac{z+2}{z-1}$. (ii)Construct a linear transformation that maps the line $x + y = 1$ on z −plane onto the circle $\|w\| = 1$ on w −plane.	**20**	**CO4**
Q11	Consider $f(z) = \frac{e^{\frac{1}{z}}}{1-\cos z}$. (i) Determine all the singularities of $f(z)$. (ii) Discuss the nature of singularity $z = 0$ of $f(z)$. (iii) Let C is the circle $z = e^{i2\theta}$ where $\theta \in [0,2\pi)$ $$\oint_C z^2 f(z)dz$$ (iv) Find the order of poles at $z = 2\pi k, k \in \mathbb{Z}\backslash\{0\}$. **OR** Find the principal value of the real integral $$\int_{-\infty}^{\infty} \frac{\sin x}{x(x^2-x+2)}\, dx\ ,$$ by clearly showing how the value of the integral $\int \frac{e^{iz}}{z(z^2-z+2)}dz \longrightarrow 0$ along the semicircular arc in upper half complex plane.	**20**	**CO3**