Name: Enrolment No:	

UPES

End Semester Examination, May 2025

Course: Advanced Engineering Mathematics II **Semester: II**
Program: B. Tech. SoAE **Time: 03 hrs.**
Course Code: MATH1065 **Max. Marks: 100**

Instructions: Attempt all questions from Section A, Section B and Section C. There are internal choices in Questions 9 and 10. Use of a scientific calculator is permitted.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's one third rule by taking $h = 0.25$.	**4**	**CO1**
Q 2	Determine constant b such that the function $u(x, y) = e^{bx} \cos 5y$ is harmonic.	**4**	**CO2**
Q 3	Discuss the nature of the singularity of the function $f(z) = \frac{e^{2z}-1}{z}$ at the point $z = 0$.	**4**	**CO2**
Q 4	Compute the Laplace transform of the function $\frac{\sin at}{t}$ by using the result $L\left[\frac{\sin t}{t}\right] = \tan^{-1}\frac{1}{s}$.	**4**	**CO4**
Q 5	Classify the partial differential equation $$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$$	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	By means of Lagrange's interpolation formula, prove that $$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$	**10**	**CO1**
Q 7	If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z, if $3u + v = 3 \sin x \cosh y + \cos x \sinh y$.	**10**	**CO2**

Q 8	Show that $x = 0$ and $x = -1$ are singular points of the following differential equation $$x^2(x+1)^2\frac{d^2y}{dx^2} + (x^2-1)\frac{dy}{dx} + 2y = 0$$ where the first is irregular and the other is regular.	**10**	**CO3**
Q 9	Using unit step function, find the Laplace transform of the following function: $$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$ **OR** Find the inverse Laplace transform of $\frac{s+1}{s^2-6s+25}$.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Apply the method of separation of variables to obtain the solution of the equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ for the case of wave motion only. A tightly stretched string with fixed end points $x = 0$ and $x = \pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity, $$\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.03\sin x - 0.04\sin 3x,$$ then find the displacement $y(x,t)$ at any point of string at any time t. **OR** Obtain the solution of the equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ for case of wave motion only using the method of separation of variables. If a string of length l is initially at rest in equilibrium position and each of its points is given the velocity, $$\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\frac{\pi x}{l},$$ find the displacement $y(x,t)$ at any point of string at any time t.	**20**	**CO5**
Q 11	(i) Apply Laplace transform to solve the differential equation: $$\frac{d^2x}{dt^2} + 9x = \cos 2t,\ x(0) = 1,\ x\left(\frac{\pi}{2}\right) = -1.$$ (ii) Obtain the Fourier Series expansion of the function $f(x) = x$ in the interval $(0, 2\pi)$.	**10+10**	**CO4**