Name:

Enrolment No:

UPES

End Semester Examination, May 2025

Course: Mathematical Physics II **Semester: II**
Program: B.Sc. (Hons.) Physics **Time : 03 hrs.**
Course Code: PHYS1034 **Max. Marks: 100**

Instructions: *Read the questions carefully and provide needed details only.*
Non-programmable Scientific calculator is allowed.

SECTION A
(5Q×4M = 20 Marks)
Answer all questions

S. No.		Marks	CO
Q 1	Show that $(\mathbb{C}; +)$ is an infinite Abelian group. Where, $\mathbb{C}$ is a set of complex numbers.	**4**	**CO1**
Q 2	Prove that $\frac{d}{dx}(x^{-n}.J_n) = -x^{-n} J_{n+1}$. Where J_n represents Bessel Function.	**4**	**CO3**
Q 3	Evaluate $\int_0^\infty \frac{x^a}{a^x}\, dx$ using Gamma function.	**4**	**CO3**
Q 4	A die is thrown 8 times. Calculate the probability that 3 will show at least seven times.	**4**	**CO2**
Q 5	Define error function (Probability integral) and complementary error function.	**4**	**CO1**

SECTION B
(4Q×10M = 40 Marks)
Q6, Q7 & Q8 are compulsory; there is an internal choice for Q9

Q 6	Show that $\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}\omega & 0\\0 & \omega^2\end{pmatrix}, \begin{pmatrix}\omega^2 & 0\\0 & \omega\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & \omega^2\\\omega & 0\end{pmatrix}, \begin{pmatrix}0 & \omega\\\omega^2 & 0\end{pmatrix}\right\}$, where $\omega^3 = 1, \omega \neq 1$ form a group with respect to matrix multiplication.	**10**	**CO1**

Q 7	Establish the relation between Beta function and Gamma function as $$\beta(m,n) = \frac{\Gamma m.\Gamma n}{\Gamma(m+n)}$$	**10**	**CO2**
Q 8	Verify that $J_n\,(x)$ is the coefficient of z^n in the expansion of $e^{\frac{x}{2}(z-\frac{1}{z})}$.	**10**	**CO1**
Q 9	Prove that the Rodrigue's formula for Legendre function is $$\int_{-1}^{+1} x^m\,P_n(x)\,dx = 0,$$ where m, n are positive integers and $m < n$, **OR** Demonstrate that $P_n(x)$ is the coefficient of z^n in the expansion of $(1-2xz+z^2)^{-\frac{1}{2}}$ in ascending powers of z.	**10**	**CO3**
SECTION-C **(2Q×20M = 40 Marks)** **Q10 is compulsory; there is an internal choice for Q11**			
Q 10	Apply Power Series method to solve the Bessel's differential equation $$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2-n^2)y = 0.$$	**20**	**CO4**
Q 11	A rod of length l with insulated sides is initially at a uniform temperature u. Its ends are suddenly cooled to 0° C and are kept at that temperature. Apply this information to prove that the temperature function $u(x,t)$ is given by $$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}.e^{-\frac{c^2\pi^2n^2t}{l^2}}$$ where b_n is determined from the equation. **OR** The vibrations of an elastic string is governed by the partial differential equation	**20**	**CO4**

	$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$ The length of the string is π and the ends are fixed. The initial velocity is zero and initial deflection is $u\,(x, 0) = 2(\sin x + \sin 3x)$. Apply, these boundary conditions to determine the deflection $u\,(x, t)$ of the vibrating string for $t \geq 0$.		