Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES
End Semester Examination, May 2025

Course: Calculus **Semester: II**
Program: B.Sc. (H) Physics/Chemistry/Geology by Research **Time : 03 hrs.**
Course Code: MATH1030 **Max. Marks: 100**

Instructions: All questions are compulsory. Scientific calculator is allowed.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Check Rolle's theorem is applicable or not for the function $f(x) = 1-(x-1)^{\frac{2}{3}}, 0 \le x \le 2.$ If not, justify the reason.	**4**	**CO1**
Q 2	Prove that the length of the subnormal at any point of the curve $x^2y^2 = a^2(x^2-a^2)$ very inversely as the cube of its abscissa.	**4**	**CO2**
Q 3	If $z^3 - x\,z - y = 0$, prove that $\frac{\partial^2 z}{\partial x \partial y} = -\frac{3\,z^2 + x}{(3z^2 - x)^3}.$	**4**	**CO2**
Q 4	Evaluate $\iint \sqrt{x^2+y^2}\,dx\,dy$, over the region bounded by $x^2+y^2 = 4$ and $x^2+y^2 = 9$.	**4**	**CO3**
Q 5	Using the definition of Beta function, prove that $B(u,v) = 2\int_0^{\pi/2} sin^{2u-1}\theta cos^{2v-1}\theta d\theta.$	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $y = \log\left(x + \sqrt{x^2+1}\right)$, prove that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$ Hence show that $y_{2n}(0) = 0$ and $y_{2n+1}(0) = (-1)^n(1^2+3^2+5^2 \ldots (2n-1)^2).$	**10**	**CO2**

Q 7	If $u = \frac{1}{3}\log\left(\frac{x^3+y^3}{x^2+y^2}\right)$, find the value of (i) $x\,\frac{\partial u}{\partial x} + y\,\frac{\partial u}{\partial y}$ (ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\,\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.	**10**	**CO1**
Q 8	Verify for the spherical coordinate transformation $x = r\sin\theta\, cos\phi$, $y = r\sin\theta sin\phi$, $z = r\cos\theta$, Jacobian $J = r^2\sin\theta$.	**10**	**CO4**
Q 9	Change the order of integration and hence evaluate $$\int_0^1\int_0^{\sqrt{1-y^2}} \frac{\cos^{-1}x}{\sqrt{1-x^2}\sqrt{1-x^2-y^2}}\,dx\,dy.$$ OR Change the order of integration and hence evaluate $$\int_0^1\int_x^{\frac{1}{x}} \frac{y}{(1+xy)^2(1+y^2)}\,dx\,dy.$$	**10** **10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	State and prove the relation between Beta and Gamma functions	**20**	**CO4**
Q 11	. Evaluate $\iiint x^2+y^2+z^2 dxdydz$, over the region bounded by the surfaces $xy = 4, xy = 9, yz = 1, yz = 4, zx = 25, zx = 49$. OR Evaluate $\iiint x^2+y^2 dx\,dy\,dz$, over the region bounded by the paraboloid $x^2+y^2 = 3z$ and the plane $z = 3$.	**20**	**CO3**