Name:

Enrolment No:

UPES

End Semester Examination, May 2025

Course: Analytical Geometry **Semester: II**

Program: B.Sc. (H) Mathematics By Research **Time : 03 hrs.**

Course Code: MATH 1069 **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Derive the equation of the ellipse whose focus is at $(-1,1)$, directrix $x-y+3=0$ and eccentricity is $1/2$.	**4**	**CO1**
Q2	Compute the points of intersection of the line $\frac{x+3}{4}=\frac{y+4}{3}=\frac{z-8}{-3}$ and the sphere $x^2+y^2+z^2+2x-10y=23$.	**4**	**CO2**
Q3	Determine the equation of the straight line which passes through the intersection of the straight lines $2x-y+1=0$, $x+y-7=0$ and is perpendicular to the straight line $6x-7y+8=0$.	**4**	**CO1**
Q4	Convert the spherical coordinates $(8,\frac{\pi}{3},\frac{\pi}{6})$ to both rectangular and cylindrical coordinates.	**4**	**CO2**
Q5	Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the equation for the circle that touches the axis of y at a distance $+4$ from the origin and cuts off an intercept 6 from the axis of x.	**10**	**CO1**
Q7	Derive the equation of the cone whose vertex is the point $(1,1,0)$ and whose guiding curve is $y=0; x^2+z^2=4$.	**10**	**CO2**

Q8	Let S_1 and S_2 be two spheres with centers at (0,0,0) and (1,1,0), and radii r_1 and r_2, respectively. Determine the angle at which the two spheres intersect, in terms r_1 and r_2.	**10**	**CO2**
Q9	Let S_1 and S_2 be two spheres in coordinate space. Prove that if S_1 and S_2 have a nonempty intersection, then the set of intersection points forms a circle. **OR** Prove that the perpendiculars drawn from the origin to the tangent planes to the cone $ax^2 + by^2 + cz^2 = 0$ lie on the cone $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$.	**10** **10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Define the notion of a tangent line to a conicoid. Determine whether the line $\frac{-(x+5)}{3} = y - 4 = \frac{z-11}{7}$ is a tangent to the conicoid $12x^2 - 17y^2 + 7z^2 = 7$.	**20**	**CO3**
Q 11	Show that the plane $3x + 12y - 6z - 17 = 0$ touches the conicoid $3x^2 - 6y^2 + 9z^2 + 17 = 0$ and find the point of contact. **OR** Prove that from any given point six normal can be drawn to a central conicoid.	**20**	**CO3**