Name: **Enrolment No:**	

UPES

End Semester Examination, May 2025

Course: Real analysis I — **Semester : II**

Program: B.Sc. (H) Mathematics by Research — **Time : 03 hrs.**

Course Code: MATH1068 — **Max. Marks: 100**

Instructions: Attempt all questions. There is an internal choice in Q 9 and Q 11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	State and prove the Archimedean property of $\mathbb{R}$.	**4**	**CO1**
Q 2	Define uniform continuous function. Is $f(x) = x^3$ uniform continuous on $\mathbb{R}$.	**4**	**CO3**
Q 3	Find the supremum and infimum of the following sets, if they exist: (1) $A = \{x \in \mathbb{R} : 2x + 5 > 0\}$ (2) $B = \left\{x \in \mathbb{R} : x < \frac{1}{x}\right\}$	**4**	**CO1**
Q 4	Use limit theorems to show that $$\lim_{n\to\infty} \frac{2n}{n^2 + 1} = 0.$$	**4**	**CO2**
Q 5	Find the interior points of the set of natural numbers $\mathbb{N}$. Is this an open set? Justify your answer.	**4**	**CO1**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Using definition of limit of a sequence, show that $$\lim_{n\to\infty} \frac{2n + 10}{n + 5} = 2.$$	**10**	**CO2**
Q 7	Define Cauchy sequence and show that $x_n = 1 + (-1)^n$ is not a Cauchy sequence.	**10**	**CO2**
Q 8	Show that finite set has no limit point. Does every infinite set have a limit point. Justify your answer.	**10**	**CO1**

Q 9	Define Lipschitz function. Show that $f(x) = x^2$ is not a Lipschitz function on $(2, \infty)$. OR Let $f(x) = cos\frac{1}{x^2}$ and $g(x) = sin\frac{1}{x}$. Using sequential criterion of limit, show that $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ does not exist.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Check the uniform continuity of the following functions on the set A: (1) $f(x) = \sin\left(\frac{1}{x}\right), A = [0, \infty)$ (2) $f(x) = \frac{1}{x^2}, A = (0, \infty)$ (3) $f(x) = \frac{1}{1+x^2},\ A = \mathbb{R}$ (4) $f(x) = \frac{1}{x},\ A = (0, \infty)$ (5) $f(x) = x^2,\ A = [2,5]$	**20**	**CO3**
Q 11	Show that a sequence $\{x_n\}$ of real numbers is convergent if and only if it is Cauchy. OR Justify your answer with example in the following cases: (a) Every monotonic sequence is a convergent sequence. (b) Every bounded sequence is a Cauchy sequence. (c) Every divergent sequence is unbounded. (d) Sequence having a unique limit point is a Cauchy sequence. (e) Every sequence has a convergent subsequence.	**20**	**CO2**