Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Discrete Mathematics | **Semester: VII**
Program: B.Sc. (Hons.) Physics by Research | **Time : 03 hrs.**
Course Code: MATH4013 | **Max. Marks: 100**

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. N		Marks	CO
Q 1	List all primes less than or equal to 50.	**4**	**CO4**
Q 2	Consider the real numbers $\mathbb{R}$ with the usual order $\preccurlyeq$. Let $A = \{x: x \in \mathbb{Q} \text{ and } 5 < x^3 < 27\}$. i) Is A bounded above or below? ii) Does sup (A) or inf (A) exist?	**4**	**CO2**
Q 3	Solve $6\,x \equiv 15 \pmod{21}$.	**4**	**CO4**
Q 4	Let gcd (a, b) = 1, show that gcd $(a + b, a - b)$ = 1 or 2.	**4**	**CO4**
Q 5	Define complemented Lattice with suitable example.	**4**	**CO2**

SECTION B
(4Qx10M= 40 Marks)

S. N		Marks	CO
Q 6	Given, set $A = \{a, b, c\}$. Give an example of a relation R defined on the set A, which is: i) reflexive and transitive but not symmetric ii) symmetric and transitive but not reflexive iii) reflexive and symmetric but not transitive	**10**	**CO1**
Q 7	Find a number x such that $x \equiv 3 \pmod{11}, x \equiv 5 \pmod{19}$, and $x \equiv 10 \pmod{29}$.	**10**	**CO4**
Q 8	Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1} + f_{n-2}, n \geq 2$ with the initial condition $f_0 = 0, f_1 = 1$.	**10**	**CO3**

Q 9	In a renowned software development company of 240 computer programmers, 102 employees are proficient in Java, 86 in C#, 126 in Python, 41 in C# and Java, 37 in Java and Python, 23 in C# and Python, and just 10 programmers are proficient in all three languages. How many computer programmers are there who are not proficient in any of these three languages? **OR** Find the minimum number of elements that one needs to take from the set $A = \{1, 2, 3, ..., 9\}$ to be sure that 2 of the numbers add up to 10.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**a)** Let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$, and $C = \{x, y, z\}$. Consider the following relations R and S from A to B and from B to C, respectively. $R = \{(1, b), (2, a), (2, c)\}$ and $S = \{(a, y), (b, x), (c, y), (c, z)\}$. i) Find the composition relation RoS. ii) Find the matrices M_R, M_S and M_{RoS} of the respective relations R, S, and RoS. iii) Check whether $M_{RoS} = M_R M_S$. **b)** Consider the function *f*: *R*→ *R* and *g*: *R*→ *R* defined by $f(a) = 2a + 1$, $g(b) = \frac{b}{3}$. Verify that $(gof)^{-1} = f^{-1} o g^{-1}$.	**20**	**CO1**
Q 11	**a)** Prove the following proposition (for $n \geq 0$): $P(n): 1 + 2 + 2^2 + 2^3 + \cdots + 2^n = 2^{n+1} - 1.$ **b)** Find the generating function for the sequence 3, -3, 3, -3, …. **OR** Use generating function technique to solve the recurrence relation $a_{n+2} - 2\, a_{n+1} + a_n = 2^n, \quad a_0 = 2, a_1 = 1.$	**20**	**CO3**