Name:

Enrolment No:

UPES

End Semester Examination, December 2024

Course: Monte Carlo Methods **Semester: V**

Program: B.Sc. Physics **Time: 03 hrs.**

Course Code: PHYS3029P **Max. Marks: 100**

Instructions:

(a) Attempt the questions as per given instructions.

(b) Nonprogrammable scientific calculator is allowed.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	A call center receives phone calls according to a Poisson process with an average rate of 20 calls per hour. Calculate the probability that exactly 4 calls are received in a 10-minute period.	**04**	**CO3**
Q 2	Suppose that $P\{X = a\} = p$, $P\{X = b\} = 1 - p$. (a) Show that $\frac{X-b}{a-b}$ is a Bernoulli random variable. (b) Find variance of X.	**04**	**CO2**
Q 3	Define simple linear congruential method and use it to generate 3 random numbers with $X_0 = 27, a = 17, c = 43,$ and $m = 100$.	**04**	**CO1**
Q 4	Find the probability of randomly drawing two aces in succession from an ordinary deck of 52 playing cards if we sample (a) Without replacement, (b) With replacement.	**04**	**CO2**
Q 5	What are some of the characteristics of pseudo-random number generator?	**04**	**CO1**

SECTION B

(4Qx10M= 40 Marks)

There is internal choice in question no. - 9

Q 6	A Markov chain has 4-states and the following transition matrix: 0.2 0.3 0.2 0.3 0.1 0.2 0.3 0.4 0.4 0.1 0.2 0.3 0.2 0.4 0.1 0.3	**10**	**CO1**

	a) Draw the state transition diagram for this chain. b) Determine whether this Markov chain is irreducible and aperiodic.		
Q 7	Explain Monte Carlo simulation of radioactive decay.	**10**	**CO3**
Q 8	If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute (a) $P\{X > 5\}$; (b) $P\{4 < X < 16\}$; (c) $P\{X > 16\}$; (d) $P\{X < 8\}$.	**10**	**CO2**
Q 9	If the joint density function of X_1 and X_2 is given by $f(x_1, x_2) = \begin{cases} 1, & 0 < x_1 < 1, \quad 0 < x_2 < 1, \\ 0, & otherwise. \end{cases}$ Find (a) the joint density of $Y = X_1 + X_2$ and $Z = X_2$; (b) the marginal density of Y. **OR** If X and Y are independent Poisson random variables with respective parameters α_1 and α_2, compute the distribution of $X + Y$.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)** **There is internal choice in question no. - 11**			
Q 10	Define Birth-death process with the help of its state transition diagram. Use pure birth process to prove that the number of births in the interval $(0, t)$ follows Poisson distribution.	**20**	**CO3**
Q 11	Generate a random variable X which follows normal distribution with mean, μ and variance σ^2 by using ratio of uniform method. **OR** Use inverse CDF method to generate a random variable X having probability density function $f(x) = \begin{cases} 2x, & 0 \le x_1 < 1, \\ 0, & otherwise. \end{cases}$ Write an algorithm for the same.	**20**	**CO1**