Name: Enrolment No:	UPES — UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Finite Element Methods **Semester: V**

Program: B.Sc. (Hons) Mathematics **Time: 03 hrs.**

Course Code: MATH3041 **Max. Marks: 100**

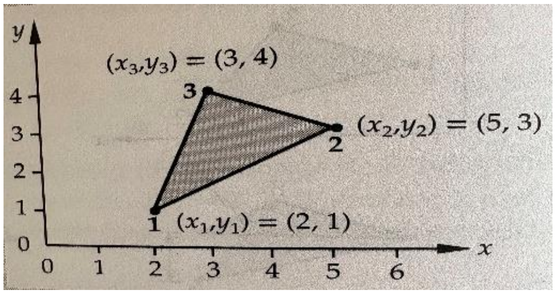
Instructions: Attempt all questions from Section A, Section B and Section C. There are internal choices in Questions 9 and 10. Use of a scientific calculator is permitted.

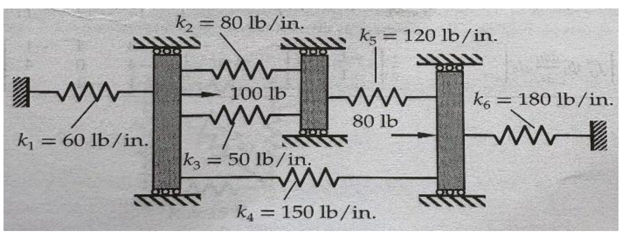
SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	The Poisson equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -1$ defined in domain D where $D = \{(x, y);\ -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$ and homogeneous Dirichlet boundary conditions are prescribed on the boundary. Find two basis functions.	**4**	**CO5**
Q 2	Consider the differential equation $-\frac{d^2u}{dx^2} - u + x^2 = 0$ with the boundary conditions $u(0) = 0$ and $u(1) = 0$. The bilinear form and linear functional associated with this problem are: $B(w, u) = \int_0^1 \left(\frac{dw}{dx}\frac{du}{dx} - wu\right) dx,\quad l(w) = -\int_0^1 wx^2 dx$, where w is the weight function. Obtain the quadratic functional $J(u)$.	**4**	**CO4**
Q 3	Derive the first order approximation from Lagrange's interpolation formula.	**4**	**CO4**
Q 4	Give an example of an eigenvalue problem in a differential equation.	**4**	**CO5**
Q 5	Using approximate solution, if Re is the residual of a differential equation. What is difference between Least square method and Collocation method?	**4**	**CO1**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Construct the weak form and the associated quadratic functional of the following nonlinear differential equation: $$-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \text{ for } 0 < x < 1;$$ $$\left[u\frac{du}{dx}\right]_{x=0} = 0,\quad u(1) = \sqrt{2}$$	**10**	**CO3**

Q 7	Determine the linear interpolation functions for the linear triangular element as shown in Figure 1.  Figure 1: Triangular element	**10**	**CO5**
Q 8	Find the Lagrange's interpolating polynomial from the following data: <table><tr><td>x:</td><td>0</td><td>1</td><td>2</td><td>5</td></tr><tr><td>$f(x)$:</td><td>2</td><td>3</td><td>12</td><td>147</td></tr></table>	**10**	**CO4**
Q 9	Solve the boundary value problem $\frac{d^2u}{dx^2} - u = x,\ 0 \leq x \leq 1,\ \ u(0) = 1, u(1) = 3$ using the approximation function $\bar{u}(x) = a_1 x(x-1) + a_2 x^2(x-1) + 2x + 1$ by Galerkin's method. **OR** Solve the boundary value problem $\frac{d^2u}{dx^2} + u = x,\ 0 \leq x \leq 1,\ \ u(0) = 0, u(1) = 0$ using the approximation function $w(x) = x(1-x)(a_1 + a_2 x)$ by the Ritz method.	**10**	**CO2**

SECTION-C (2Qx20M=40 Marks)			
Q 10	Consider the problem of finding the transverse deflection of a cantilever beam under a uniform transverse load of intensity q_0 per unit length and subjected to the point load F_0 and bending moment M_0 at the free end. The governing equations according to Euler-Bernoulli beam theory are $\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) - q_0 = 0$ for $0 < x < L,\ EI > 0$ $w(0) = \left[\frac{dw}{dx}\right]_{x=0} = 0,\ \left[EI\frac{d^2w}{dx^2}\right]_{x=L} = -M_0,\ \left[\frac{d}{dx}\left(EI\frac{d^2w}{dx^2}\right)\right]_{x=L} = F_0,$ where $EI > 0$ is the flexural rigidity of the beam (E is the modulus of elasticity and I is the second moment of area about y axis). Determine an n parameter Ritz solution using the basis function $\varphi_n(x) = x^{n+1}$. Find the Ritz solution for $n = 1$ also. **OR** Solve the boundary value problem $\frac{d^2u}{dx^2} + u = x,\ \ 0 < x < 1,\ u(0) = 0,$ $u(1) = 0$ using the Ritz finite element method with linear piecewise polynomials for three elements of equal length.	**20**	**CO3**
Q 11	Consider the system of linear elastic springs shown in Figure 2. Assemble the element equations to obtain the force displacement relations for the entire system. Use boundary conditions to write the condensed equations for the unknown displacement and forces.  Figure 2: System of linear elastic springs	**20**	**CO5**