Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: **Linear and Non-linear Programming** **Semester : V**

Program: **B. Sc. (H) Mathematics** **Time : 03 hrs.**

Course Code: MATH3032 **Max. Marks: 100**

Instructions: Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 10 have internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate the problem as a linear programming problem (LPP) model. Do not solve it.	**4**	**CO1**
Q 2	Determine all basic feasible solutions for the system of equations: $$2x_1 + x_2 + 4x_3 = 11 \text{ and } 3x_1 + x_2 + 5x_3 = 14.$$	**4**	**CO1**
Q 3	Construct the dual of the following LPP: $$Max.\, z = x_1 - x_2 + 3x_3$$ subject to the constraints $x_1 + x_2 + x_3 \leq 10,\quad 2x_1 - x_3 \leq 2,\quad 2x_1 - 2x_2 + 3x_3 \leq 6,\quad$ and $x_1, x_2, x_3 \geq 0.$	**4**	**CO2**
Q 4	Define assignment problem. What is the mathematical formulation of an assignment problem?	**4**	**CO2**
Q 5	State Kuhn-Tucker necessary and sufficient conditions in non-linear programming.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Show that the set $S = \{(x_1, x_2, x_3): 2x_1 - x_2 + x_3 \leq 4\} \subset R^3$, is a convex set.	**10**	**CO1**

<table>
<tr><td>Q 7</td><td>By applying the graphical method, solve the LPP:
$$Max. z = 60x_1 + 40x_2$$
subject to the constraints:
$$x_1 + 2x_2 \leq 12,\ 2x_1 + x_2 \leq 12,\ x_1 + \frac{5}{4}x_2 \geq 5 \text{ and } x_1, x_2 \geq 0.$$</td><td>10</td><td>CO1</td></tr>
<tr><td>Q 8</td><td>Determine an initial basic feasible solution to the following transportation problem using Vogel's method where O_i and D_j represents the i^{th} origin and j^{th} destination respectively.
<table>
<tr><th>Origin\Destination</th><th>D_1</th><th>D_2</th><th>D_3</th><th>D_4</th><th>Supply</th></tr>
<tr><td>O_1</td><td>11</td><td>13</td><td>17</td><td>14</td><td>250</td></tr>
<tr><td>O_2</td><td>16</td><td>18</td><td>24</td><td>10</td><td>300</td></tr>
<tr><td>O_3</td><td>21</td><td>24</td><td>13</td><td>10</td><td>400</td></tr>
<tr><td>Demand</td><td>200</td><td>225</td><td>275</td><td>250</td><td>950</td></tr>
</table>
OR
A supervisor wishes to assign five jobs among the five machines in a machine shop. Any one of the jobs can be processed completely by any one of the machines as given below:
<table>
<tr><th>Jobs\Machines</th><th>M_1</th><th>M_2</th><th>M_3</th><th>M_4</th><th>M_5</th></tr>
<tr><td>J_1</td><td>13</td><td>8</td><td>16</td><td>18</td><td>19</td></tr>
<tr><td>J_2</td><td>9</td><td>15</td><td>24</td><td>9</td><td>12</td></tr>
<tr><td>J_3</td><td>12</td><td>9</td><td>4</td><td>4</td><td>4</td></tr>
<tr><td>J_4</td><td>6</td><td>12</td><td>10</td><td>8</td><td>13</td></tr>
<tr><td>J_5</td><td>15</td><td>17</td><td>18</td><td>12</td><td>20</td></tr>
</table>
The assignment of jobs to machines be on a one-to-one basis. Assign the jobs to machines so that the total cost is minimum. Find the minimum total cost.</td><td>10</td><td>CO2</td></tr>
<tr><td>Q 9</td><td>Find the value of x_1 and x_2 for the optimum solution of the following non-linear programming problem:
$$Min. z = f(x_1, x_2) = 3e^{2x_1+1} + 2e^{x_2+5}$$
subject to the constraints:
$$x_1 + x_2 = 7, \text{ and } x_1, x_2 \geq 0.$$</td><td>10</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>Apply simplex method to solve the following LPP:
$$Max. z = 3x_1 + 5x_2 + 4x_3$$
subject to the constraints
$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$
$$\text{and } x_1, x_2, x_3 \geq 0.$$</td><td>20</td><td>CO2</td></tr>
</table>

	OR Use penalty (Big-M) method to solve the following LPP: $Max. z = x_1 + 2x_2 + 3x_3 - x_4$ subject to the constraints $x_1 + 2x_2 + 3x_3 = 15$ $2x_1 + x_2 + 5x_3 = 20$ $x_1 + 2x_2 + x_3 + x_4 = 10$ and $x_1, x_2, x_3, x_4 \geq 0$.		
Q 11	Solve the non-linear programming problem: $Optimize\ z = 4{x_1}^2 + 2{x_2}^2 + {x_3}^2 - 4x_1x_2$ subject to the constraints: $x_1 + x_2 + x_3 = 15$, $2x_1 - x_2 + 2x_3 = 20$, $x_1, x_2, x_3 \geq 0$.	**20**	**CO3**