Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Course: Ordinary Differential Equations (Major) **Semester: III**
Program: B.Sc. (H) Mathematics by Research **Time: 03 hrs**
Course Code: MATH2048 **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:

1. Mention Name and Roll No. at the top of the question paper.
2. Attempt all questions. There are internal choices for question 9 and question 11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Solve the following differential equation by using suitable method: $$\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$$	**4**	**CO1**
Q 2	Solve the following differential equation $$(D^3+3D^2+3D)y = 0 \quad D \equiv \frac{d}{dx}$$	**4**	**CO3**
Q 3	In a certain country, the population gets tripled in 5 years and after 10 years, the population is 20,000. Find the number of people initially being living in the city.	**4**	**CO2**
Q 4	Explain homogeneous differential equation. Hence, solve the following differential equation: $$(4y+3x)dy + (y-2x)dx = 0$$	**4**	**CO1**
Q 5	Find the particular integral of the following differential equation $$(D^2-3D+4)y = e^{2x}\sin 2x; \quad D \equiv \frac{d}{dx}$$	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Solve the following differential equation using method of undetermined coefficients: $$(D^2-2D+3)y = x^3+\sin x; \quad D \equiv \frac{d}{dx}$$	**10**	**CO3**

Q 7	Solve the following differential equation $$(D-1)^2(D^2+1)^2 y = \sin x; \quad D \equiv \frac{d}{dx}$$	**10**	**CO3**
Q 8	Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei in a sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 200 years. Then evaluate the following: (i) What percentage of the original radioactive nuclei will remain after 1000 years? (ii) In how many years will only one– fourth of the original number remain?	**10**	**CO2**
Q 9	Reduce $y = 2px + y^2p^3$ to Clairaut's form by using a suitable substitution and hence find its general and singular solution. **OR** Find the solution of the following differential equation, which passes through (0,0): $$\frac{dy}{dx} + \frac{1}{x}(\sin 2y) = x^2\cos^2 y$$	**10**	**CO1**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Explain the Lotka-Volterra Model. Further, let a population of rabbits (prey) grows at a rate of $\alpha = 0.5$ per year in the absence of predators, and the foxes (predators) have a death rate of $\gamma = 0.3$ per year. The rate at which foxes kill rabbits is $\beta = 0.01$ per rabbit per fox, and the growth rate of the fox population per rabbit eaten is $\delta=0.02$. Find the equilibrium populations of rabbits and foxes.	**20**	**CO4**
Q. 11	Solve the following nonhomogeneous linear differential equation by using variation of parameter method: $$(D^2+1)y = \sec x\,; \quad D \equiv \frac{d}{dx}$$ **OR** Solve the following Cauchy-Euler differential equation: $$(x^4D^4 + 6x^3D^3 + 4x^2D^2 - 2xD - 4)y = 2\cos(\log x);$$ $$D \equiv \frac{d}{dx}$$	**20**	**CO3**