


Name:			
Enrolment No:			
<b>UPES</b> <b>End Semester Examination, December 2024</b>			
<b>Course: Integral Calculus</b> <b>Program: B.Sc. (H) Mathematics by Research</b> <b>Course Code: MATH 2056</b>		<b>Semester: III</b> <b>Time : 03 hrs.</b> <b>Max. Marks: 100</b>	
<b>Instructions: Attempt all the questions. There are internal choices in the question number 6 and 10.</b>			
<b>SECTION A</b> <b>(5Qx4M=20Marks)</b>			
S. No.		Marks	CO
Q 1	For any function $\phi(x)$ , show that $\int_a^b \phi(x) dx = -\int_b^a \phi(x) dx$ .	4	CO1
Q 2	Define the improper integrals of second kind with the help of an example.	4	CO2
Q 3	Prove that $\Gamma(1+n)\Gamma(1-n) = \frac{n\pi}{\sin n\pi}$ .	4	CO2
Q 4	Show that the area of the circle $x^2 + y^2 = a^2$ is $\pi a^2$ using the double integration.	4	CO3
Q 5	For any position vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $ \vec{r}  = r$ . Show that $\nabla r^n = nr^{n-2}\vec{r}$ .	4	CO4
<b>SECTION B</b> <b>(4Qx10M= 40 Marks)</b>			
Q 6	If $u_n = \int_0^{\frac{\pi}{2}} x^n \sin mx dx$ , prove that $u_n = \frac{n\pi^{n-1}}{m^2 2^{n-1}} - \frac{n(n-1)}{m^2} u_{n-2},$ if $m$ is of the form $4r + 1$ .  <b>OR</b> Find the reduction formula for $\int e^{ax} \sin^n bx dx$ .	10	CO1
Q 7	Check whether the integral $\int_0^{\infty} e^{-a^2 x^2} \cos bx dx$ converges absolutely or not?	10	CO2
Q 8	Evaluate $\iint_R y^2 dx dy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$ .	10	CO3

Q 9	A particle moves along the curve $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ , where $t$ is the time. Find the magnitude of the tangential components of its acceleration at $t = 2$ .	10	CO4
<b>SECTION-C</b> <b>(2Qx20M=40 Marks)</b>			
Q 10	Transform $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sqrt{\left(\frac{\sin \phi}{\sin \theta}\right)} d\phi d\theta$ by the substitution $x = \sin \phi \cos \theta$ , $y = \sin \phi \sin \theta$ and show that its value is $\pi$ . <b>OR</b> Find the volume cut off from the sphere $x^2 + y^2 + z^2 = a^2$ by the cylinder $x^2 + y^2 = ax$ .	20	CO2
Q 11	State and prove the Green's theorem for a plane.	20	CO4