Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Programme Name: B.Sc. Hons. Mathematics / B.Sc. Hons. Mathematics by Research — **Semester: I**

Course Name : Linear Algebra I — **Time : 03 hrs**

Course Code : MATH 1057 — **Max. Marks: 100**

Nos. of page(s) : 2

Instructions: All questions are compulsory. There are internal choices in Q6 and Q11.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Check if the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$ is diagonalizable. Justify your answer.	**4**	**CO1**
Q 2	What is the algebraic multiplicity and geometric multiplicity of an eigenvalue? Hence, mention the criterion for the diagonalizability of a square matrix.	**4**	**CO1**
Q 3	State the necessary and sufficient condition for a subset of a vector space to be a subspace. Is the set $\{\mathbf{0}\}$ containing only the zero vector, a subspace of the vector space? Explain.	**4**	**CO2**
Q 4	If $\{v_1, v_2, v_3, v_4\}$ is a basis for a vector space V and U is a subset of V such that $v_1, v_2 \in U$, and $v_3, v_4 \notin U$. Prove that U is a subspace of V. If not, give a counter example.	**4**	**CO2**
Q 5	Examine the linearity of the transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$, such that $T(x, y, z) = (x, y, xz)$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Let $U_1 = \{(x, y, 0) \in \mathbb{R}^3 : x, y \in \mathbb{R}\}$, $U_2 = \{(0, 0, z) \in \mathbb{R}^3 : z \in \mathbb{R}\}$, $U_3 = \{(0, y, y) \in \mathbb{R}^3 : y \in \mathbb{R}\}$ be subspaces of $\mathbb{R}^3$. Then, show that the sum of these subspaces, $U_1 + U_2 + U_3$ is not a direct sum. **OR**	**10**	**CO2**

	Show that the intersection of any finite number of subspaces of a vector space is also a subspace.		
Q 7	What is the order of the matrix representation of the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$. Let $M: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined as $$M(x, y, z) = (3x + 2y - 4z, x - 5y + 3z).$$ Find the matrix of M with respect to the basis, $$S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}.$$	**10**	**CO3**
Q 8	State the Rank Nullity Theorem. Let $AX = B$ denote the matrix form of a system of m linear equations in n unknowns. Then the matrix A may be viewed as a linear mapping $A: \mathbb{R}^n \to \mathbb{R}^m$. What will be the dimension of kernel of A, if the rank of the matrix is r.	**10**	**CO3**
Q 9	Determine whether the linear transformation $T: P_3 \to M_{22}$ is an isomorphism or not, where $$T(a + bx + cx^2 + dx^3) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$ Here, P_3 is a vector space of all the polynomials with degree less than or equal to 3, and M_{22} is a vector space of all the matrices of order 2×2. Also find the rank of T and nullity of T.	**10**	**CO3**
SECTION-C (2Qx20M=40 Marks)			
Q 10	Let V be a vector space spanned by $\boldsymbol{f_1} = \sin x$ and $\boldsymbol{f_2} = \cos x$. a) Show that $\boldsymbol{g_1} = 2\sin x + \cos x$, and $\boldsymbol{g_2} = 3\cos x$ form a basis for V. b) Find the transition matrix from $B' = \{\boldsymbol{g_1}, \boldsymbol{g_2}\}$ to $B = \{\boldsymbol{f_1}, \boldsymbol{f_2}\}$. c) Find the transition matrix from B to B'. d) Compute the coordinate vector $(\boldsymbol{h})_B$, where $\boldsymbol{h} = 2\sin x - 5\cos x$. Use it to find $(\boldsymbol{h})_{B'}$.	**20**	**CO2**
Q 11	Let $T: \mathbb{R}^{\mathbf{2}} \to \mathbb{R}^{\mathbf{3}}$ be a linear transformation defined by $$T(x_1, x_2) = (x_1 + 3x_2, x_1 - x_2, x_1).$$ Find the matrix representation of T with respect to the ordered basis $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$ and $\{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\}$, where $\boldsymbol{u}_1 = (1, 2)$, $\boldsymbol{u}_2 = (3, 1)$, and $\boldsymbol{b}_1 = (1, 0, 0)$, $\boldsymbol{b}_2 = (1, 1, 0)$, $\boldsymbol{b}_3 = (1, 1, 1)$. Hence find the dimension of the kernel of T and comment if the transformation is one-one and onto. **OR** If the matrix representation of a transformation $T: \mathbb{R}^{\mathbf{3}} \to \mathbb{R}^{\mathbf{3}}$ with respect to the standard basis is given by $$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}.$$ Find the transformation $T(x, y, z)$. Check if the transformation is invertible. If yes, find $T^{-1}(x, y, z)$.	**20**	**CO3**