Name: Enrolment No:	

UPES

END Semester Examination, December 2024

Programme Name : M.Sc. (Mathematics) **Semester : I**
Course Name : Real Analysis **Time : 03 hrs**
Course Code : MATH7040 **Max. Marks: 100**
Nos. of page(s) : 02

Instructions: All questions are compulsory. There is an internal choice in Q9 and Q11.

SECTION A
(4 Marks * 5 = 20 Marks)
Answer all questions

S. No.		Marks	CO
Q 1	Consider the series of functions $$\sum_{n=1}^{\infty}\frac{\cos(nx^2)}{n(n+1)},x\in\mathbb{R}$$ Identify the uniform convergence of the series.	4	CO1
Q 2	Find lower Riemann sum and upper Riemann sum of the function $$f(x)=\begin{cases}-5 & if\ x\in\mathbb{Q}\\ 5 & if\ x\notin\ \mathbb{Q}\end{cases}.$$	4	CO2
Q 3	Determine the radius of convergence and the exact interval of convergence of the power series $$\sum\frac{(n+1)x^n}{(n+2)(n+3)}.$$	4	CO3
Q 4	Check whether the following function is differentiable at (0,0) or not? $$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0).\end{cases}$$	4	CO4
Q 5	Let $f_n(x)$ be given by $$f_n(x)=\frac{x^{2n}}{1+x^{2n}},x\ \in\mathbb{R};n\ \in\mathbb{N}$$ find the limit function $f(x)$.	4	CO1

SECTION B **(10 Marks * 4 = 40 Marks)** **Answer all questions. There is an internal choice in Q9.**			
Q 6	By giving an example, show that the conditions of Young's theorem and Schwarz's theorem are sufficient but not necessary.	**10**	**CO4**
Q 7	Test the convergence of the series $$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4+\cdots$$ Also, find the domain of convergence.	**10**	**CO3**
Q8	Investigate for the pointwise and uniform convergence of the sequence of functions given by $f_n(x)=\frac{n^2x}{1+n^3x^2}$; $x\in[0,1]$.	**10**	**CO1**
Q 9	Show that $f(x)=k$ (constant) is Riemann-integrable in $[0,\frac{\pi}{2}]$ **OR** Prove that a continuous function $f:[a,b]\to R$ on a compact interval is Riemann integrable.	**10**	**CO2**
SECTION C **(20 Marks * 2 = 40 Marks)** **Answer all questions. There is an internal choice in Q11.**			
Q 10	If $$\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$$ Then, find the value of $$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}.$$	**20**	**CO3**
Q 11	Show that for the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by $$f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^4} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$$ the directional derivatives exist for all directions at the point (0,0) but the function is not continuous at (0,0). ***OR*** Determine whether the function $$f(x,y)=\begin{cases}\frac{x^3-y^3}{x^3+y^3} & if\ (x,y)\neq(0,0)\\ 0 & if\ (x,y)=(0,0)\end{cases}$$ is differentiable or not at (0,0). Also find $f_x(x,y)$ and $f_y(x,y)$ at (0,0).	**20**	**CO4**