Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Course: Mathematical Statistics **Semester: I**
Program: M.Sc. Mathematics **Time: 03 hrs.**
Course Code: MATH7039 **Max. Marks: 100**

Instructions: Attempt all the questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let X be a gamma random variable with parameters α and λ. Calculate Var[X].	**04**	**CO1**
Q 2	The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill is 3.42 grams with a standard deviation of 0.68 grams. Construct a 99% confidence interval for the true mean weight loss of such grinding balls under the stated conditions.	**04**	**CO3**
Q 3	The time for a super glue to set can be treated as a random variable having a normal distribution with mean 30 seconds. Find its standard deviation if the probability is 0.2 that it will take on a value greater than 39.2 seconds.	**04**	**CO1**
Q 4	Coefficient of correlation between X and Y is 0.3. Their covariance is 9 and variance of X is 16. Find the standard deviation of Y.	**04**	**CO2**
Q 5	Suppose we have a random sample of size $2n$ from a population denoted by X, E[X]= μ and Var[X] = σ^2. Let $$\bar{X}_1 = \frac{1}{2n}\sum_{i=1}^{2n} X_i \text{ , and } \bar{X}_2 = \frac{1}{n}\sum_{i=1}^{n} X_i,$$ be two estimators of μ. Which one is better? Provide a proper justification.	**04**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6 Suppose that the cumulative distribution function of the random variable X is given by $F(x) = 1 - e^{-x^2}, x > 0$. Evaluate

(a) $P(X > 2)$;
(b) $P(1 < X < 3)$;
(c) $E[X]$.

10 | **CO1**

sQ 7 Fit a simple linear regression model (Y on X) as well as (X on Y) to the data on salt concentration and road way area denoted by Y and X respectively, given as follows:-

Y	3.8	5.9	14.1	10.4	14.6	14.5	15.1	11.9	15.5	9.3
X	0.19	0.15	0.57	0.4	0.7	0.67	0.63	0.47	0.75	0.6

10 | **CO2**

Q8 Find the correlation coefficient for the given data

x_i	5	6	7	8	10	12
y_i	15	18	24	26	27	32

10 | **CO2**

Q9 If the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} xy, & 0 < x < 1, \quad 0 < y < 2, \\ 0, & otherwise, \end{cases}$$

(a) Find the marginal density function of both X and Y;
(b) Are X and Y independent;
(c) Compute the conditional density X given that $Y = y$;

OR

With $\Phi(x)$ being the probability that a normal random variable with mean 0 and variance 1 is less than x, which of the following are true:

(a) $\Phi(-x) = \Phi(x)$
(b) $\Phi(-x) + \Phi(x) = 1$
(c) $\Phi(-x) = 1/\Phi(x)$

10 | **CO1**

SECTION-C
(2Qx20M=40 Marks)

Q 10 Consider the probability density function

$$\mathrm{f(x)} = \frac{1}{\theta} x e^{-x^2/\theta}, 0 \le x < \infty, 0 < \theta < \infty.$$

(a) Show that $E[X^2] = 2\theta$. Use this information to construct an unbiased estimator for θ.
(b) Find the maximum likelihood estimator of θ.

20 | **CO3**

Q 11	Suppose it is known that the IQ scores of a certain population of adults are approximately normally distributed with standard deviation, 15. A simple random sample of 25 adults drawn from this population had a mean IQ score of 105. On the basis of these data can we conclude that the mean IQ score for population is not 100? Let the probability of committing a Type-I error be 0.05. Calculate P-value and justify your conclusion. **OR** The following data are the oxygen uptakes (milliliters) during incubation of a random sample of 15 cell suspensions: 14.0, 14.1, 14.5, 13.2, 11.2, 14.0, 14.1, 12.2, 11.1, 13.7, 13.2, 16.0, 12.8, 14.4, 12.9 Do these data provide sufficient evidence at the 0.5 level of significance that the population mean is not 12 ml? Assume normality.	**20**	**CO4**