Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Course: Advanced Abstract Algebra **Semester : I**
Program: M.Sc. Mathematics **Time : 03 hrs.**
Course Code: MATH7038 **Max. Marks: 100**

Instructions: 1. This question paper contains 11 questions.
2. Attempt all questions from Section A (each carrying 4 marks).
3. Attempt all questions from Section B (each carrying 10 marks). Question 9 has an internal choice.
4. Attempt all questions from Section C (each carrying 20 marks). Question 11 has an internal choice.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find the irreducibility of the polynomial $f(x) = x^3 + 2x^2 + 5x + 1$ over $\mathbb{Q}$.	**4**	**CO2**
Q 2	State the First Isomorphism Theorem for modules.	**4**	**CO3**
Q 3	Find the splitting field of $f(x) = x^2 + 1$ over $\mathbb{Q}$.	**4**	**CO4**
Q 4	Define algebraic extension of a field and provide an example.	**4**	**CO4**
Q 5	State Fundamental Theorem of Galois Theory.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Let F be a field of order 4096. Find the number of proper subfields of F.	**10**	**CO4**
Q 7	Prove that a map $\phi: M \to N$ is an R-module homomorphism if and only if $\phi(rx + y) = r\phi(x) + \phi(y)$ for all $x, y \in M$ and for all $r \in R$.	**10**	**CO3**
Q 8	Let $\phi : M \to N$ be an R-module homomorphism. Then, prove that $\phi(M)$ is a submodule of N.	**10**	**CO3**
Q 9	If a group of order $5^2.2.3$ has no normal subgroups, then find the number of Sylow 5-subgroups. **OR** If $\sigma = (1\ 5\ 2\ 4)$ and $\tau = (1\ 4\ 2)(3\ 5)$, then find the order of $\sigma\tau$ in S_5.	**10**	**CO1**

SECTION-C
(2Qx20M=40 Marks)

Q 10	Let $N_1, N_2, \ldots N_k$ be submodules of the R-module M. Then, prove that the following are equivalent.	**20**	**CO3**

	i. $N_j \cap (N_1 + N_2 + \cdots N_k) = 0$ for all $j \in \{1,2, \dots, k\}$. ii. Every $x \in N_1 + N_2 + \cdots N_k$ can be written uniquely in the form $a_1 + a_2 + \cdots + a_k$ with $a_i \in N_i$.		
Q 11	If G is a group of order pq, where p and q are primes, $p < q$ and p does not divide $q - 1$, then prove that G is cyclic. **OR** Prove that i. $Z(G)$, center of a group G is a subgroup of G and it is normal. ii. every subgroup of an abelian group is normal.	**20**	**CO1**