Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, December 2024

Course: Mathematical Physics - I **Semester: I**

Program: BSc (H) Physics/BSc (H) Physics by Research **Time : 03 hrs.**

Course Code: PHYS 1037 **Max. Marks: 100**

Instructions:

1. *All questions are mandatory in Section A.*
2. *Internal choices are given in questions #8 and #10 of Sections B and C respectively*

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	What is the physical meaning of gradient of a scalar field? If you are given a scalar field $\varphi(x,y) = x^2y - y^3$, find the gradient of this scalar field at $(-1,2,5)$.	**4**	**CO1**
Q2	When do you call a vector field solenoidal and irrotational? Explain the physical meaning of the terms "Solenoidal" and "Conservative".	**4**	**CO1**
Q3	Verify whether the matrix is orthogonal or not. $$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$$	**4**	**CO2**
Q4	Find the Eigen values of the following matrix: $$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$	**4**	**CO2**
Q5	List the properties of Dirac Delta function.	**4**	**CO2**

SECTION B

(4Qx10M= 40 Marks)

Q6	Find the complete solution of the following 2nd order linear differential equation: $$(D^2 - 4D + 3)y = 2x\,e^{2x} + 3e^x \cos 2x$$ where $D = \frac{d}{dx}$ and $D^2 = \frac{d^2}{dx^2}$ are 1st and 2nd order differential operators, respectively.	**10**	**CO3**
Q7	a) Solve the following differential equation: (**5 Marks**) $$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$$ b) Find the solution of following differential equation: (**5 Marks**)	**10**	**CO3**

	$$\frac{dy}{dx} + xy = x^3y^3$$		
Q8	A vector field $\vec{F}$ is defined as $$\vec{F} = 2xz\,\hat{\imath} + 2yz^2\hat{\jmath} + (x^2 + 2y^2z - 1)\hat{k}$$ Calculate $\vec{\nabla} \times \vec{F}$ and deduce that $\vec{F}$ can be written as $\vec{F} = \overrightarrow{\nabla\varphi}$, where φ is a scalar field. Determine the form of φ. **OR** Find the directional derivative of A^2, where $\vec{A} = xy^2\hat{\imath} + zy^2\hat{\jmath} + xz^2\hat{k}$, at the point (2,0,3) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 14$ at the point (3,2,1).	**10**	**CO4**
Q9	Find the matrix which diagonalizes the following matrix $$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$ Also, write the diagonal matrix.	**10**	**CO2**
SECTION-C (2Qx20M=40 Marks)			
Q10	The vector field $\vec{Q}$ is given as $$\vec{Q} = [3x^2(y+z) + y^3 + z^3]\hat{\imath} + [3y^2(z+x) + z^3 + x^3]\hat{\jmath} + [3z^2(x+y) + x^3 + y^3]\hat{k}$$ Show that $\vec{Q}$ is a conservative field, construct its potential function (φ) and hence evaluate the integral $$J = \int \overrightarrow{\nabla\varphi} . \overrightarrow{dr}$$ Along any the line connecting the point A at (1,-1,1) to B at (2,1,2). **OR** A vector force field $\vec{F}$ is defined in Cartesian coordinates by $$\vec{F} = F_0\left[\left(\frac{y^3}{3a^3} + \frac{y}{a}e^{xy/a^2} + 1\right)\hat{\imath} + \left(\frac{xy^2}{a^3} + \frac{x+y}{a}e^{xy/a^2}\right)\hat{\jmath} + \frac{z}{a}e^{xy/a^2}\hat{k}\right]$$ Use Stokes' theorem to calculate $$\oint_L \vec{F} \cdot \overrightarrow{dr}$$ where L is the boundary of the rectangle ABCD given by A = (1,1,0), B = (1,1,0), C = (1,3,0) and D = (0,3,0). [Hint: Do not directly calculate the line integral; use Stokes' theorem to calculate it, in which you need to calculate $\int_S \vec{\nabla} \times \vec{F} . \overrightarrow{dS}$, where the area enclosed in L.	**20**	**CO4**

Q11	a) Use the Divergence theorem to evaluate $$\int_S \vec{F}.\overrightarrow{dS}$$ where $\vec{F} = 4x^3\hat{\imath} - x^2y\hat{\jmath} + x^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = a^2$ bounded by planes $z = 0$ and $z = b$. [**13 Marks**] [*Hint*: Do not directly calculate the surface integral; use Divergence theorem to calculate, which means you need to evaluate $\int_V \vec{\nabla} \cdot \vec{F}\, dV$, where V is the volume enclosed by the cylinder, to get the surface integral]. b) For the scalar field $\varphi = x^2y + yz$ at the point (1,2,-1), find the rate of change in the direction $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. At this same point, what is the greatest possible rate of change with distance and in which direction does it occur? [**7 Marks**]	**20**	**CO4**