Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

Programme Name: B.Sc. Hons. Physics / B.Sc. Hons. Physics by Research — **Semester: I**

Course Name : Linear Algebra I (Minor) — **Time : 03 hrs**

Course Code : MATH 1057 — **Max. Marks: 100**

Nos. of page(s) : 2

Instructions: All questions are compulsory. There are internal choices in Q6 and Q11.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	If A and B are square matrices, there exists an invertible matrix P such that $B = P^{-1}AP$. If X is the eigenvector of B corresponding to an eigenvalue λ, what will be the eigenvector of A corresponding to the same eigenvalue. Prove it.	**4**	**CO1**
Q 2	Check for the diagonalizability of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$ without finding the eigenvectors. Explain the reason.	**4**	**CO1**
Q 3	Let $V = \mathbb{R}^3$. Check whether W is a subspace of V, where $W = \{(a, b, c) \in \mathbb{R}^3 : a^2 + b^2 + c^2 \leq 1\}$.	**4**	**CO2**
Q 4	Find the basis and dimension of a subspace $\{(x, x, y) \in \mathbb{R}^3 : x, y \in \mathbb{R}\}$ of $\mathbb{R}^3$.	**4**	**CO2**
Q 5	If A is an $m \times n$ matrix, what is the largest possible value for its rank and the smallest possible value for its nullity?	**4**	**CO3**
SECTION B **(4Qx10M= 40 Marks)**			
Q 6	Write the necessary and sufficient condition for the sum of two subspaces of a vector space to be a direct sum. Let U be the xy −plane and W be the yz −plane in $\mathbb{R}^3$. Can we write every vector in $\mathbb{R}^3$ as the sum of a vector in U and a vector in W? Justify your answer. **OR** Suppose $U = \{(x, x, y, y) \in \mathbb{R}^4 : x, y \in \mathbb{R}\}$. Find a subspace W of $\mathbb{R}^4$, such that $\mathbb{R}^4 = U \oplus W$.	**10**	**CO2**

Q 7	Consider the basis $S = \{v_1,\ v_2\}$ for $\mathbb{R}^2$, where $v_1 = (1,1)$ and $v_2 = (1,0)$, and let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation for which $T(v_1) = (1,-2)$ and $T(v_2) = (-4,1)$. Find a formula for $T(x,y)$, and use that to find $T(5,-3)$. Also find the formula for $T^{-1}(x,y)$, if exists.	**10**	**CO3**
Q 8	State the Rank Nullity Theorem, and use it to find the nullity of T, if a) $T: \mathbb{R}^5 \to P_5$ has rank 3, b) $T: P_4 \to P_3$ has rank 1, where P_n is a vector space of all the polynomial with degree less than or equal to n, defined over the real field.	**10**	**CO3**
Q 9	Define Isomorphism on vector spaces. Let $T: P_2 \to M_{22}$ be a linear transformation defined by $$T(p(x)) = \begin{bmatrix} p(0) & p(1) \\ p(1) & p(0) \end{bmatrix},$$ where P_2 is a vector space of all the polynomials with degree less than or equal to 2, and M_{22} is a vector space of all the matrices of order 2×2. Is T a linear transformation. Justify your answer.	**10**	**CO3**
SECTION-C (2Qx20M=40 Marks)			
Q 10	Consider the bases $B = \{\boldsymbol{p_1}, \boldsymbol{p_2}\}$, and $B' = \{\boldsymbol{q_1}, \boldsymbol{q_2}\}$ for a vector space of all the polynomial with degree less than or equal to 1, P_1, where $$\boldsymbol{p_1} = 6 + 3x,\ \boldsymbol{p_2} = 10 + 2x,\ \boldsymbol{q_1} = 2,\ \boldsymbol{q_2} = 3 + 2x$$ a) Find a transition matrix from B' to B. b) Find a transition matrix from B to B'. c) Compute the coordinate vector $(\boldsymbol{p})_B$, where $\boldsymbol{p} = -4 + x$, and use it to compute $(\boldsymbol{p})_{B'}$.	**20**	**CO2**
Q 11	Find the matrix representation of each linear transformation, $T: \mathbb{R}^3 \to \mathbb{R}^3$, relative to the standard basis of $\mathbb{R}^3$. Also find the nullity of T in each case. a) $T(x,y,z) = (x,y,0)$ b) $T(x,y,z) = (z, y+z, x+y+z)$ c) $T(x,y,z) = (2x - 7y - 4z, 3x + y + 4z, 6x - 8y + z)$ **OR** The matrix representation of a linear transformation $T: P_2 \to P_2$ with respect to the standard basis is given by $$\begin{bmatrix} 1 & -5 & 25 \\ 0 & 3 & -30 \\ 0 & 0 & 9 \end{bmatrix}.$$ Find a formula for the transformation $T(a + bx + cx^2)$. Check if T is a one-one onto map. If yes, find $T^{-1}(a + bx + cx^2)$.	**20**	**CO3**