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UPES
End Semester Examination, December 2024

Course: Introductory Mathematical Physics **Semester: I**
Program: BSC-H-Geology, BSC-H-Mathematics & BSC-H-Mathematics by Research, BSC-H-Chemistry
Time: 03 hours
Course Code: PHYS1031 **Max. Marks: 100**

Instructions:
(1) All questions are compulsory. Section A has 5 questions; Section B has 4 questions with an internal choice in one question; Section C has 2 questions with an internal choice in one question.
(2) Use of scientific calculators is permitted.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	List any two properties of the Dirac-Delta function.	4	CO1
Q2	If any two complex numbers $a + ib$ and $c + id$ are equal, show that $a = c$ and $b = d$.	4	CO2
Q3	Solve the following integral: $\int_{-\infty}^{\infty} e^{-5t} \delta(t - 2) dt$.	4	CO2
Q4	Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = 0$.	4	CO3
Q5	Determine the constant 'a' such that the vector $\vec{A} = (x + 3y)\hat{i} + (2y + 3z)\hat{j} + (x + az)\hat{k}$ is solenoidal.	4	CO3

SECTION B
(4Qx10M= 40 Marks)
There is an internal choice in Q8.

Q6	Prove that $\Gamma(n + 1) = n\Gamma(n)$ where Γ is the Gamma function.	10	CO2
Q7	Derive a relation between the Beta function and Gamma function.	10	CO2
Q8	Show that $e^{i\theta} = \cos \theta + i \sin \theta$. <p style="text-align: center;">OR</p> Express in polar form the complex number: $1 - \sqrt{2} + i$.	10	CO3
Q9	Find the Fourier Series representation of the function $f(x) = x$ where $0 < x < 2\pi$.	10	CO3

SECTION-C
(2Qx20M=40 Marks)
There is an internal choice in Q11.

Q10	Use the technique of separation of variables to find the solution $u(x, t)$ to the following Partial Differential Equation (k is a constant): $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \cdot \frac{\partial u}{\partial t}$	20	CO4
Q11	<p>(a) Using the Divergence Theorem, illustrate that $\iint_S \nabla \cdot (x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k}) \, dS = 6V$ where S is any closed surface enclosing the volume V.</p> <p>(b) Determine the value of: $i^{49} + i^{103} + 3$.</p> <p style="text-align: center;">OR</p> <p>(a) Evaluate $\iint_S \vec{F} \cdot \vec{dS}$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.</p> <p>(b) Solve: $\frac{1+i}{3+4i}$.</p>	15	CO4
		5	CO3
		15	CO4
		5	CO3
