Name: **Enrolment No:**	

UPES

End Semester Examination, December 2024

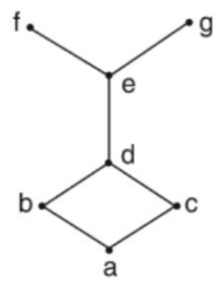
Course : Discrete Mathematics & Linear Algebra **Semester : III**
Programme : B.Sc. Computer Science **Time : 03 hrs**
Course Code : CSEG2057 **Max. Marks: 100**
Nos. of page(s) : 03

Instructions: Answer ALL the questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.	**4**	**CO4**
Q 2	Show that the relation $R = \{(a, b) \in Z \times Z: a \ divides\ b\ in\ Z\}$ is not partial order on Z .	**4**	**CO3**
Q 3	Define a field. Give suitable examples.	**4**	**CO5**
Q 4	Write the converse, inverse and contrapositive of the following statements: i) The home team wins whenever it is raining. ii) If n is greater than 3, then n^2 is greater than 9.	**4**	**CO2**
Q 5	Let G be a group. If $a, b \in G$ such that $a^4 = e$, the identity element of G and $ab = ba^2$, prove that $a = e$.	**4**	**CO5**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Define lattice and determine whether the poset represented by the given Hasse diagram is a lattice or not.  	**10**	**CO3**

Q 7	Prove that the set $G = \{1,2,3,4,5,6\}$ forms an Abelian group with respect to multiplication modulo 7.	**10**	**CO5**
Q 8	Solve the recurrence relation $a_{n+2} + 2a_{n+1} - 15a_n = 6n + 10$ that satisfies the initial conditions $a_0 = 1, a_1 = -\frac{1}{2}$.	**10**	**CO1**
Q 9	What is the principal disjunctive normal form? Find the principal disjunctive normal form of $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$. **OR** Prove the following equivalence by using laws of propositional algebra $P \to (Q \vee R) \equiv (P \to Q) \vee (P \to R)$	**10**	**CO2**
	SECTION-C **(2Qx20M=40 Marks)**		
Q 10	(a) Let $\mathcal{F}$ be the field of complex numbers and T be the function from $\mathcal{F}^3$ into $\mathcal{F}^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2 - x_3, -x_1 - 2x_2)$. Verify that T is a linear transformation. Describe the null space of T. (b) Find the matrix of T in the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (1,0,1), \alpha_2 = (-1,2,1), \alpha_3 = (2,1,1)$, and T is the linear operator on $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$.	**20**	**CO6**
Q 11	Diagonalize the following matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ **OR** (a) Determine the value of λ such that the system of homogeneous equations $2x + y + 2z = 0;\ x + y + 3z = 0; 4x + 3y + \lambda z = 0$ has (i) trivial solution and (ii) non-trivial solution. Find the non-trivial solution. (b) Find the rank of the following matrix by reducing it to normal form	**20**	**CO4**

	$$B = \begin{bmatrix} 3 & 4 & 1 & 1 \\ 2 & 4 & 3 & 6 \\ -1 & -2 & 6 & 4 \\ 1 & -1 & 2 & -2 \end{bmatrix}.$$		