Name: **Enrolment No:**	

UPES
End Semester Examination, May 2024

Programme Name: Integrated B.Sc.-M.Sc. Mathematics — **Semester : VI**
Course Name : Topology — **Time : 03 hrs**
Course Code : MATH 3048 — **Max. Marks: 100**
Nos. of page(s) : 02

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Suppose $S \subset \mathbb{R}^2$ is a set which is the union of an open triangle and its three vertices. Determine the number of connected components in S.	**4**	**CO5**
Q2	Give an example of topological space which is T_1 but not T_2. Give reasons for your choice.	**4**	**CO5**
Q3	Determine whether the circle $S = \{(x, y) \| x^2 + y^2 = 1\}$ is homeomorphic to the interval $[0,1]$ or not. Justify your answer.	**4**	**CO4**
Q4	Suppose (M_1, τ_1) and (M_2, τ_2) are topological spaces such that $\tau_1 \neq \tau_2$. Describe explicitly the product topology $\tau_1 \times \tau_2$ on the cartesian product $M_1 \times M_2$.	**4**	**CO4**
Q5	Suggest two topologies τ_1 and τ_2 on some set M such that the (M, τ_1) is separable but (M, τ_2) is not.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Prove that the space $\mathbb{C}^n$ endowed with standard topology is connected.	**10**	**CO5**
Q7	Consider the set of real numbers $\mathbb{R}$ equipped with the standard topology. Is $\mathbb{R}$ compact? Justify your answer by choosing a suitable base for the topology.	**10**	**CO4**
Q8	Suppose (X, τ) is a topological space and $Y \subset X$. Prove that the restriction of τ on set Y defined as $\tau\|_Y = \{u \cap Y \mid u \in \tau\}$ is an induced topology on Y.	**10**	**CO3**

Q9	If X and Y are topological spaces and let $X \sim Y$ mean that X and Y are homeomorphic. Show that this relation is an equivalence relation. **OR** Let X, Y and Z are topological spaces. If $f: X \to Y$ and $g: Y \to Z$ are continuous mappings, show that $gf: X \to Z$ is also continuous.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	(a) Define uniform continuity of a function $f: X \to X$ in the metric space (X, d). If $X = \mathbb{R}$ and $d(x, y) = \|x - y\|$; $x, y \in \mathbb{R}$ is the Euclidean metric defined on it then prove or disprove that $f(x) = (x+1)^{1/2024}$ is uniformly continuous on $[-1, \infty) \subset \mathbb{R}$. (b) Is the function $f: \mathbb{R}^+ \to \mathbb{R}$ defined as $f(x) = \sin\left(\frac{1}{x}\right)$ uniformly continuous? Justify your answer by choosing appropriate sequences in $\mathbb{R}^+$to check the criterion for uniform convergence.	**20**	**CO2**
Q11	Prove that in a metric space (X, d) every closed sphere is a closed set. Also show that the union of an infinite class of closed sets is not necessarily closed. **OR** Let X be a complete metric space and let $\{F_n\}$ be a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$. Prove that $F = \cup_{n=1}^{\infty} F_n$ contains exactly one point.	**20**	**CO1**